OBJECTIVE 1

1 EXAMPLE Simplify $4\sqrt{3} + \sqrt{3}$.

$4\sqrt{3} + \sqrt{3} = 4\sqrt{3} + 1\sqrt{3}$ Both terms contain $\sqrt{3}$.

$= (4 + 1)\sqrt{3}$ Use the Distributive Property to combine like radicals.

$= 5\sqrt{3}$ Simplify.

OBJECTIVE 1

2 EXAMPLE Simplify $8\sqrt{5} - \sqrt{45}$.

$8\sqrt{5} - \sqrt{45} = 8\sqrt{5} + \sqrt{9 \cdot 5}$ — 9 is a perfect square and a factor of 45.

$= 8\sqrt{5} - \sqrt{9} \cdot \sqrt{5}$ — Use the Multiplication Property of Square Roots.

$= 8\sqrt{5} - 3\sqrt{5}$ — Simplify $\sqrt{9}$.

$= (8 - 3)\sqrt{5}$ — Use the Distributive Property to combine like terms.

$= 5\sqrt{5}$ — Simplify.

OBJECTIVE 2

3 EXAMPLE Simplify $\sqrt{5}(\sqrt{8} + 9)$.

$\sqrt{5}(\sqrt{8} + 9) = \sqrt{40} + 9\sqrt{5}$ Use the Distributive Property.

$= \sqrt{4} \cdot \sqrt{10} + 9\sqrt{5}$ Use the Multiplication Property of Square Roots.

$= 2\sqrt{10} + 9\sqrt{5}$ Simplify.

OBJECTIVE 2

4 EXAMPLE Simplify $(\sqrt{6} - 3\sqrt{21})(\sqrt{6} + \sqrt{21})$.

$(\sqrt{6} - 3\sqrt{21})(\sqrt{6} + \sqrt{21})$

$= \sqrt{36} + \sqrt{126} - 3\sqrt{126} - 3\sqrt{441}$ Use FOIL.

$= 6 - 2\sqrt{126} - 3(21)$ Combine like radicals and simplify $\sqrt{36}$ and $\sqrt{441}$.

$= 6 - 2\sqrt{9 \cdot 14} - 63$ 9 is a perfect square factor of 126.

$= 6 - 2\sqrt{9} \cdot \sqrt{14} - 63$ Use the Multiplication Property of Square Roots.

$= 6 - 6\sqrt{14} - 63$ Simplify $\sqrt{9}$.

$= -57 - 6\sqrt{14}$ Simplify.

OBJECTIVE 2

5 EXAMPLE Simplify $\frac{8}{\sqrt{7}-\sqrt{3}}$.

$= \frac{8}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$ Multiply the numerator and denominator by the conjugate of the denominator.

$= \frac{8(\sqrt{7}+\sqrt{3})}{7-3}$ Multiply in the denominator.

$= \frac{8(\sqrt{7}+\sqrt{3})}{4}$ Simplify the denominator.

$= 2(\sqrt{7}+\sqrt{3})$ Divide 8 and 4 by the common factor 4.

$= 2\sqrt{7} + 2\sqrt{3}$ Simplify the expression.

OBJECTIVE 2

6 EXAMPLE A painting has a length : width ratio approximately equal to the golden ratio $(1 + \sqrt{5}) : 2$. The length of the painting is 51 in. Find the exact width of the painting in simplest radical form. Then find the approximate width to the nearest inch.

Define: 51 = length of painting
x = width of painting

Relate: $(1 + \sqrt{5}) : 2$ = length : width

Write: $\dfrac{(1 + \sqrt{5})}{2} = \dfrac{51}{x}$

$x(1 + \sqrt{5}) = 102$ Cross multiply.

$\dfrac{x(1 + \sqrt{5})}{(1 + \sqrt{5})} = \dfrac{102}{(1 + \sqrt{5})}$ Solve for x.

OBJECTIVE 2

6 EXAMPLE (continued)

$$x = \frac{102}{(1+\sqrt{5})} \cdot \frac{(1-\sqrt{5})}{(1-\sqrt{5})}$$ Multiply the numerator and the denominator by the conjugate of the denominator.

$$x = \frac{102(1-\sqrt{5})}{1-5}$$ Multiply in the denominator.

$$x = \frac{102(1-\sqrt{5})}{-4}$$ Simplify the denominator.

$$x = \frac{-51(1-\sqrt{5})}{2}$$ Divide 102 and –4 by the common factor –2.

$x = 31.51973343$ Use a calculator.

$x \approx 32$

The exact width of the painting is $\frac{-51(1-\sqrt{5})}{2}$ inches.

The approximate width of the painting is 32 inches.