Name _____ Class _____ Date _____

Reteaching 1-5

Absolute Value Equations and Inequalities

OBJECTIVE: Solving absolute value equations **MATERIALS:** None

For every positive real number a, both a and $-a$ satisfy the equation $|x| = a$.

To solve an absolute value equation, first rewrite the equation as an equivalent equation with the absolute value expression on the left side by itself. Then rewrite this equation as a compound equality using the rule that if $|x| = a$ then $x = a$ or $x = -a$.

Example

Solve the equation $2|x - 3| + 1 = 6x + 7$. Check for extraneous solutions.

Use the properties of equality to rewrite the equation as an equivalent equation with the absolute value expression on one side by itself. Then write that equation as a compound equality and solve each resulting equation.

$$2|x - 3| + 1 = 6x + 7$$
$$2|x - 3| = 6x + 6 \quad \leftarrow \textbf{Subtract 1 from each side.}$$
$$|x - 3| = 3x + 3 \quad \leftarrow \textbf{Divide each side by 2.}$$
$$x - 3 = 3x + 3 \text{ or } x - 3 = -(3x + 3) \quad \leftarrow \textbf{Rewrite as a compound equality.}$$
$$-2x = 6 \text{ or } x - 3 = -3x - 3 \quad \leftarrow \textbf{Solve each equation.}$$
$$x = -3 \text{ or } 4x = 0$$
$$x = -3 \text{ or } x = 0$$

To check for extraneous solutions, substitute each value for x in the original absolute value equation. Any value that does not satisfy the original equation must be discarded.

Check $2|-3 - 3| + 1 \stackrel{?}{=} 6(-3) + 7$ $\quad 2|0 - 3| + 1 \stackrel{?}{=} 6(0) + 7$

$2|-6| + 1 \stackrel{?}{=} -18 + 7$ $\quad 2|-3| + 1 \stackrel{?}{=} 0 + 7$

$2(6) + 1 \stackrel{?}{=} -11$ $\quad 2(3) + 1 \stackrel{?}{=} 7$

$13 \neq -11$ $\quad 7 = 7$

The only solution is 0; -3 is an extraneous solution.

Exercises

Solve each equation. Check for extraneous solutions.

1. $|2x + 7| = 5$ **2.** $|x - 3| = -1$ **3.** $|x + 7| = 2x + 8$ **4.** $|x - 0.5| + 0.3 = 1$

5. $3|2x + 5| = 15$ **6.** $|5x - 1| + 7 = 3x$ **7.** $2|x + 1| + x = 1$ **8.** $|x + 1| = 2x$