Name _____ Class _____ Date _____

Reteaching 9-3

Rational Functions and Their Graphs

OBJECTIVE: Finding and classifying points of discontinuity

MATERIALS: None

Rational functions may have two different types of points of discontinuity.

- A hole is present at $x = a$ when a is a zero of both the numerator and the denominator.
- A vertical asymptote is present at $x = a$ when a is a zero of the denominator only.
- Find points of discontinuity before attempting to graph the function.

Example

Find and classify any points of discontinuity for $y = \frac{x^2 + x - 6}{3x^2 - 12}$.

$$y = \frac{x^2 + x - 6}{3x^2 - 12}$$

$$y = \frac{(x - 2)(x + 3)}{3(x - 2)(x + 2)} \leftarrow \textbf{Factor the numerator and denominator completely.}$$

$$y = \frac{(x - 2)(x + 3)}{3(x - 2)(x + 2)} \leftarrow \textbf{Circle common factors in the numerator and denominator to indicate holes.}$$

$$x - 2 = 0 \leftarrow \textbf{Use the Zero-Product Property to find the point of discontinuity.}$$

$$x = 2$$

$$x + 2 = 0 \leftarrow \textbf{Use the Zero-Product Property with any remaining factors in the denominator to find the asymptotes.}$$

$$x = -2$$

There is a hole at $x = 2$ and a vertical asymptote at $x = -2$.

Exercises

Find and classify any points of discontinuity.

1. $y = \frac{x}{x^2 - 9}$

2. $y = \frac{3x^2 - 1}{x^3}$

3. $y = \frac{6x^2 + 3}{x - 1}$

4. $y = \frac{5x^3 - 4}{x^2 + 4x - 5}$

5. $y = \frac{7x}{x^3 + 1}$

6. $y = \frac{12x^4 + 10x - 3}{3x^4}$

7. $y = \frac{12x + 24}{x^2 + 2x}$

8. $y = \frac{x^2 - 1}{x^2 + 3x + 2}$

9. $y = \frac{x^2 - 1}{x^2 - 2x - 3}$