

Reteaching 11-5

Geometric Series

OBJECTIVE: Finding the sum of a finite and of an infinite geometric series **MATERIALS:** None

- The sum of a finite geometric series is the sum of the terms of a geometric sequence. This sum can be found by using the formula

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \text{ where } a_1 \text{ is the first term, } r \text{ is the common ratio, and } n \text{ is the number of terms.}$$

- The sum of an infinite geometric series with $|r| < 1$ is found by using the formula $S = \frac{a_1}{1 - r}$, where a_1 is the first term and r is the common ratio. If $|r| \geq 1$, then the series has no sum.

Example

Find the sum of the first ten terms of the series

$$8 + 16 + 32 + 64 + 128 + \dots$$

$$a_1 = 8$$

$$r = \frac{16}{8} = \frac{32}{16} = \frac{64}{32} = \frac{128}{64} = 2$$

$$n = 10$$

$$S_{10} = \frac{8(1 - 2^{10})}{1 - 2}$$

$$= \frac{8(-1023)}{-1}$$

$$= 8184$$

- ← a_1 is the first term in the series.
- ← Simplify the ratio formed by any two consecutive terms to find r .
- ← n is the number of terms in the series to be added together.
- ← Substitute $a_1 = 8$, $r = 2$, and $n = 10$ into the formula for the sum of a finite geometric series.
- ← Simplify inside the parentheses.
- ← Simplify.

Exercises

Evaluate the finite series for the specified number of terms.

1. $3 + 12 + 48 + 192 + \dots; n = 6$

2. $8 + 2 + \frac{1}{2} + \frac{1}{8} + \dots; n = 5$

3. $-10 - 5 - 2.5 - 1.25 - \dots; n = 7$

4. $10 + (-5) + \frac{5}{2} + \left(-\frac{5}{4}\right) + \dots; n = 11$

Evaluate each infinite geometric series.

5. $10 + 5 + 2.5 + \dots$

6. $-1 + \frac{2}{11} - \frac{4}{121} + \dots$

7. $\frac{1}{4} + \frac{7}{32} + \frac{49}{256} + \dots$

8. $\frac{1}{2} - \frac{1}{5} + \frac{2}{25} - \dots$

9. $-\frac{1}{6} + \frac{1}{12} - \frac{1}{24} + \dots$

10. $20 + 16 + \frac{64}{5} + \dots$

11. $12 + 4 + \frac{4}{3} + \dots$

12. $\frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \dots$

13. $\frac{2}{3} + \frac{2}{15} + \frac{2}{75} + \dots$