ALGEBRA REVIEW

Basic Numerical and Operational Properties

Mr. Rivera

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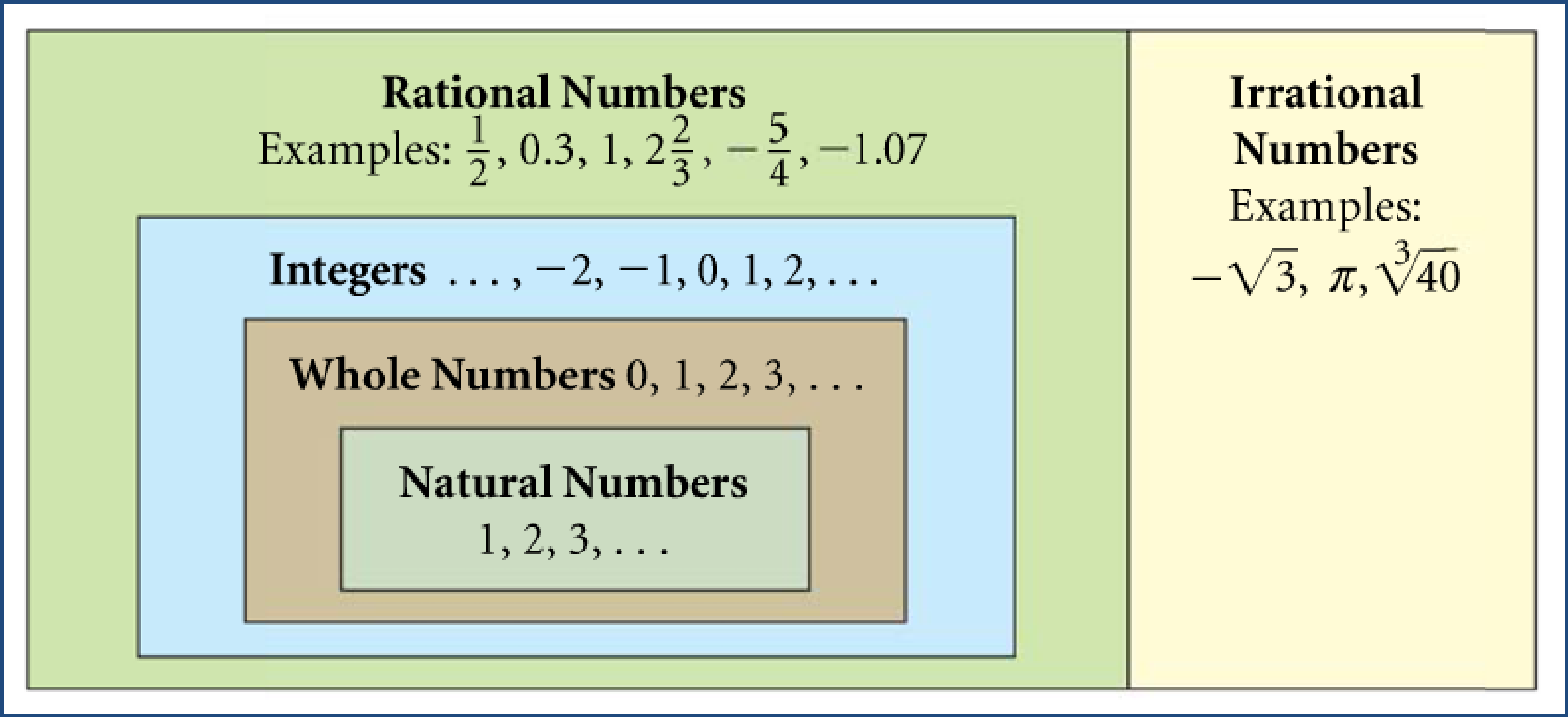
Set of Real Numbers



Mr. Rivera – Math Class

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Basic Numerical and Operational Properties

Summary **Properties of Real Numbers**

Let a, b, and c represent real numbers.

Property	Addition	Multiplication
Closure	$a + b$ is a real number.	ab is a real number.
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	$a + 0 = a, 0 + a = a$	$a \cdot 1 = a, 1 \cdot a = a$
Inverse	$a + (-a) = 0$	$a \cdot \frac{1}{a} = 1, a \neq 0$
Distributive	$a(b + c) = ab + ac$	

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Summary — Properties for Simplifying Algebraic Expressions

Let a, b, and c represent real numbers.

Definition of Subtraction	$a - b = a + (-b)$
Definition of Division	$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}, b \neq 0$
Distributive Property for Subtraction	$a(b - c) = ab - ac$
Multiplication by 0	$0 \cdot a = 0$
Multiplication by −1	$-1 \cdot a = -a$
Opposite of a Sum	$-(a + b) = -a + (-b)$
Opposite of a Difference	$-(a - b) = b - a$
Opposite of a Product	$-(ab) = -a \cdot b = a \cdot (-b)$
Opposite of an Opposite	$-(-a) = a$

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Summary **Properties of Equality**

Let a, b, and c represent real numbers.

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then b may be substituted for a in any expression to obtain an equivalent expression.

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Property	Properties of Inequalities
Let a, b, and c represent real numbers.	
Transitive Property	If $a \le b$ and $b \le c$, then $a \le c$.
Addition Property	If $a \le b$, then $a + c \le b + c$.
Subtraction Property	If $a \le b$, then $a - c \le b - c$.
Multiplication Property	If $a \le b$ and $c > 0$, then $ac \le bc$. If $a \le b$ and $c < 0$, then $ac \ge bc$. ←
Division Property	If $a \le b$ and $c > 0$, then $\frac{a}{c} \le \frac{b}{c}$. If $a \le b$ and $c < 0$, then $\frac{a}{c} \ge \frac{b}{c}$. ←

You must reverse the inequality symbol when c is negative.

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Definition	**Algebraic Definition of Absolute Value**
• If $x \geq 0$, then $\lvert x \rvert = x$.	• If $x < 0$, then $\lvert x \rvert = -x$.

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EXAMPLE **Solving Absolute Value Equations**

Solve $|2y - 4| = 12$.

$|2y - 4| = 12$

$2y - 4 = 12$ or $2y - 4 = -12$ — The value of $2y - 4$ can be 12 or -12 since $|12|$ and $|-12|$ both equal 12.

$2y = 16$ | $2y = -8$ — Add 4 to each side of both equations.

$y = 8$ or $y = -4$ — Divide each side of both equations by 2.

Check $|2y - 4| = 12$

$|2(8) - 4| \stackrel{?}{=} 12$ $\qquad$ $|2(-4) - 4| \stackrel{?}{=} 12$

$|12| = 12$ ✓ $\qquad$ $|-12| = 12$ ✓

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Definition	Extraneous Solution

An **extraneous solution** is a solution of an equation derived from an original equation that is not a solution of the original equation.

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EXAMPLE **Checking for Extraneous Solutions**

Solve $|2x + 5| = 3x + 4$.

$$|2x + 5| = 3x + 4$$

$2x + 5 = 3x + 4$ or $2x + 5 = -(3x + 4)$ **Rewrite as two equations.**

$-x = -1$ | $2x + 5 = -3x - 4$ **Solve each equation.**

$x = 1$ | $5x = -9$

$x = 1$ or $x = -\frac{9}{5}$

Check $|2x + 5| = 3x + 4$ $\qquad |2x + 5| = 3x + 4$

$|2(1) + 5| \stackrel{?}{=} 3(1) + 4$ $\qquad \left|2\left(-\frac{9}{5}\right) + 5\right| \stackrel{?}{=} 3\left(-\frac{9}{5}\right) + 4$

$|7| \stackrel{?}{=} 7$ $\qquad \left|\frac{7}{5}\right| \stackrel{?}{=} -\frac{7}{5}$

$7 = 7$ ✓ $\qquad \frac{7}{5} \neq -\frac{7}{5}$

The only solution is 1. $-\frac{9}{5}$ is an extraneous solution.

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Properties | **Absolute Value Inequalities**

Let k represent a positive real number.

$\lvert x \rvert \geq k$	is equivalent to	$x \leq -k$ or $x \geq k$.
$\lvert x \rvert \leq k$	is equivalent to	$-k \leq x \leq k$.

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EXAMPLE **Solving Absolute Value Inequalities, $|A| < b$**

Solve $3|2x + 6| - 9 < 15$. Graph the solution.

$3|2x + 6| - 9 < 15$	
$3|2x + 6| < 24$	Isolate the absolute value expression. Add 9 to each side.
$|2x + 6| < 8$	Divide each side by 3.
$-8 < 2x + 6 < 8$	Rewrite as a compound inequality.
$-14 < 2x < 2$	Solve for *x*.
$-7 < x < 1$