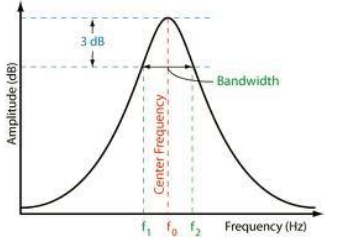
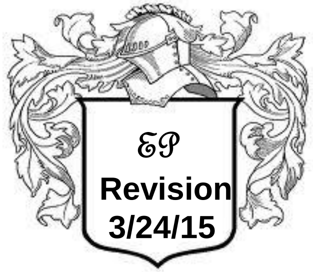
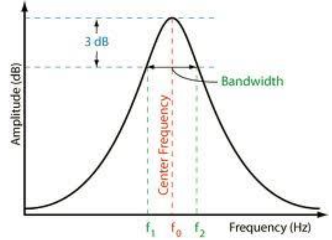
FILTERS – Transfer Functions/Bode Plots



- ❑ Frequency domain Analysis
- ❑ Transfer Functions
- ❑ Bode Plot Generation



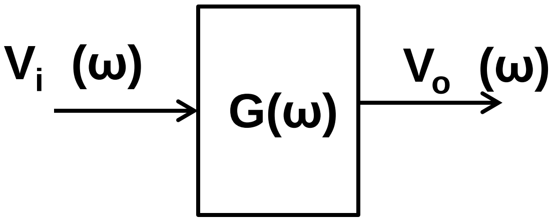
FILTERS – Transfer Functions/Bode Plots



A transfer function is defined as the ratio of the frequency domain output voltage to the frequency domain input voltage (i.e. Gain) with all initial conditions equal to zero. Transfer functions are defined only for linear systems.

Transfer functions can usually be expressed as the ratio of two polynomials in the complex variable, s (i.e. $s = j\omega$)

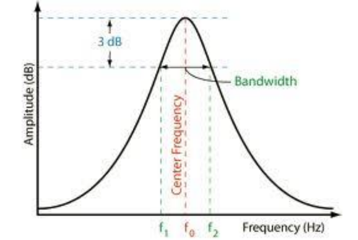
A transfer function can be factored into the following form.



$$H(s) = \frac{V_0}{V_i} = \frac{K(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ \ldots\ (s+p_n)}$$

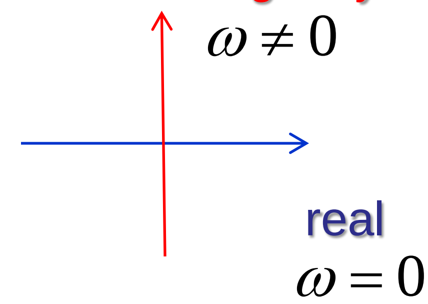
- *The roots of the numerator polynomial are called zeros*
- *The roots of the denominator polynomial are called poles*

FILTERS – Transfer Functions/Bode Plots



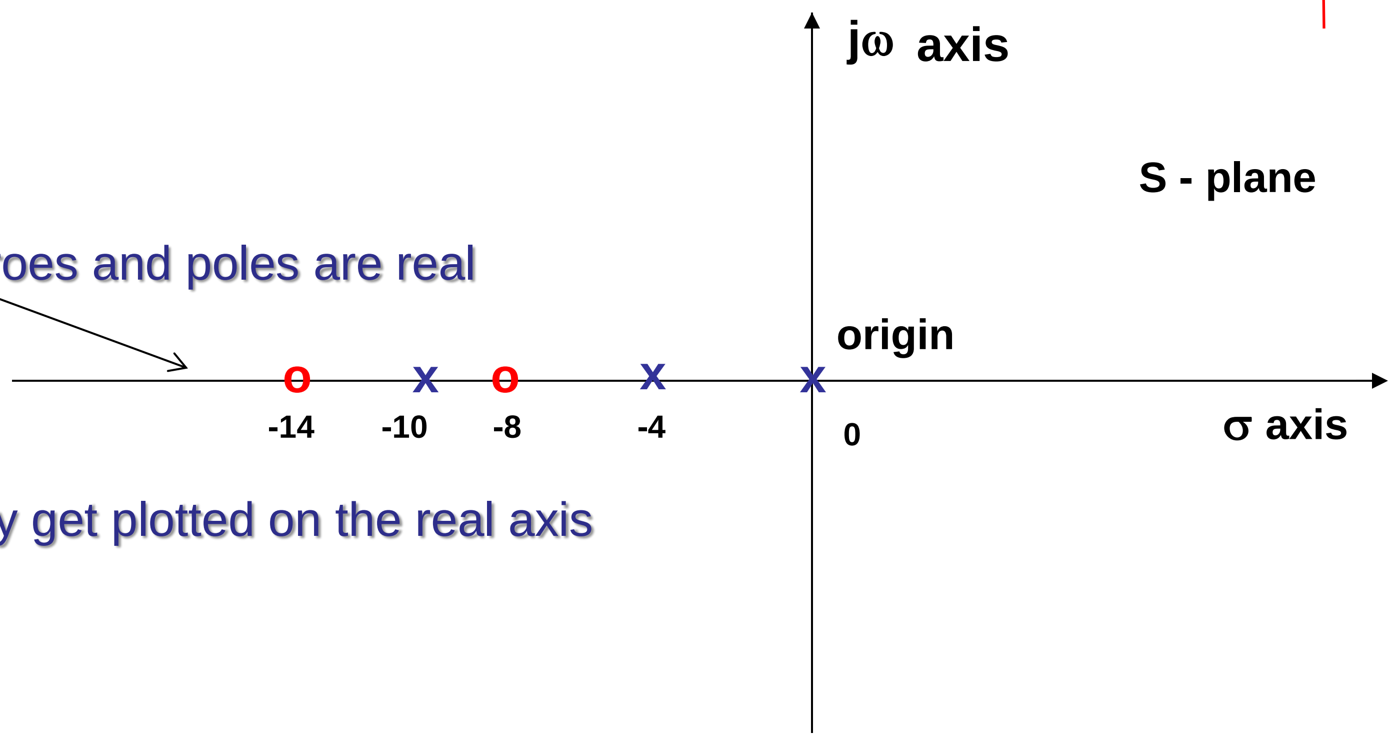
Given the transfer function. Plot the poles and zeros in the s-plane.

$$H(s) = \frac{(s+8)(s+14)}{s(s+4)(s+10)}$$



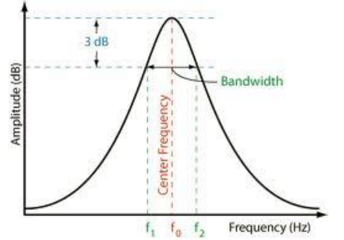
S-plane

These zeroes and poles are real



Thus, they get plotted on the real axis

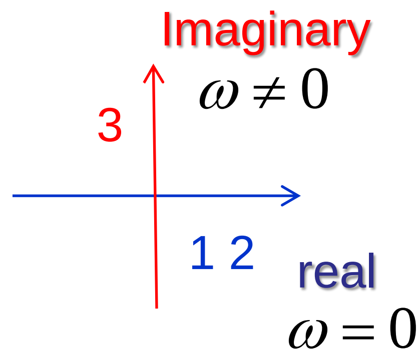
FILTERS – Transfer Functions/Bode Plots



Quadratic Equation

$$ar^2 + br + c = 0$$

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



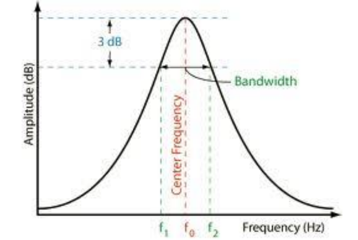
Discriminant

1. $b^2 - 4ac > 0$
2. $b^2 - 4ac = 0$
3. $b^2 - 4ac < 0$

Roots of $ar^2 + br + c = 0$
1. r_1, r_2 real and distinct 2. $r_1 = r_2 = r$ 3. r_1, r_2 complex: $\alpha \pm i\beta$

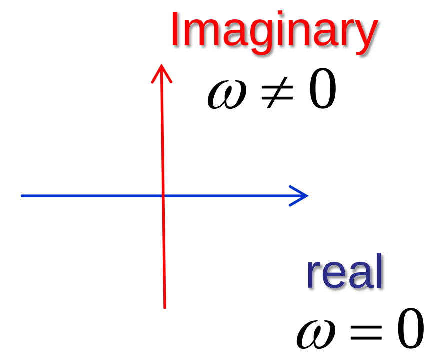
1. $s^2 + 5s + 6 = (s+2)(s+3)$
2. $(s+2)^2 = (s+2)(s+2)$
3. $s^2 + 4 = (s+2j)(s-2j)$

FILTERS – Transfer Functions/Bode Plots

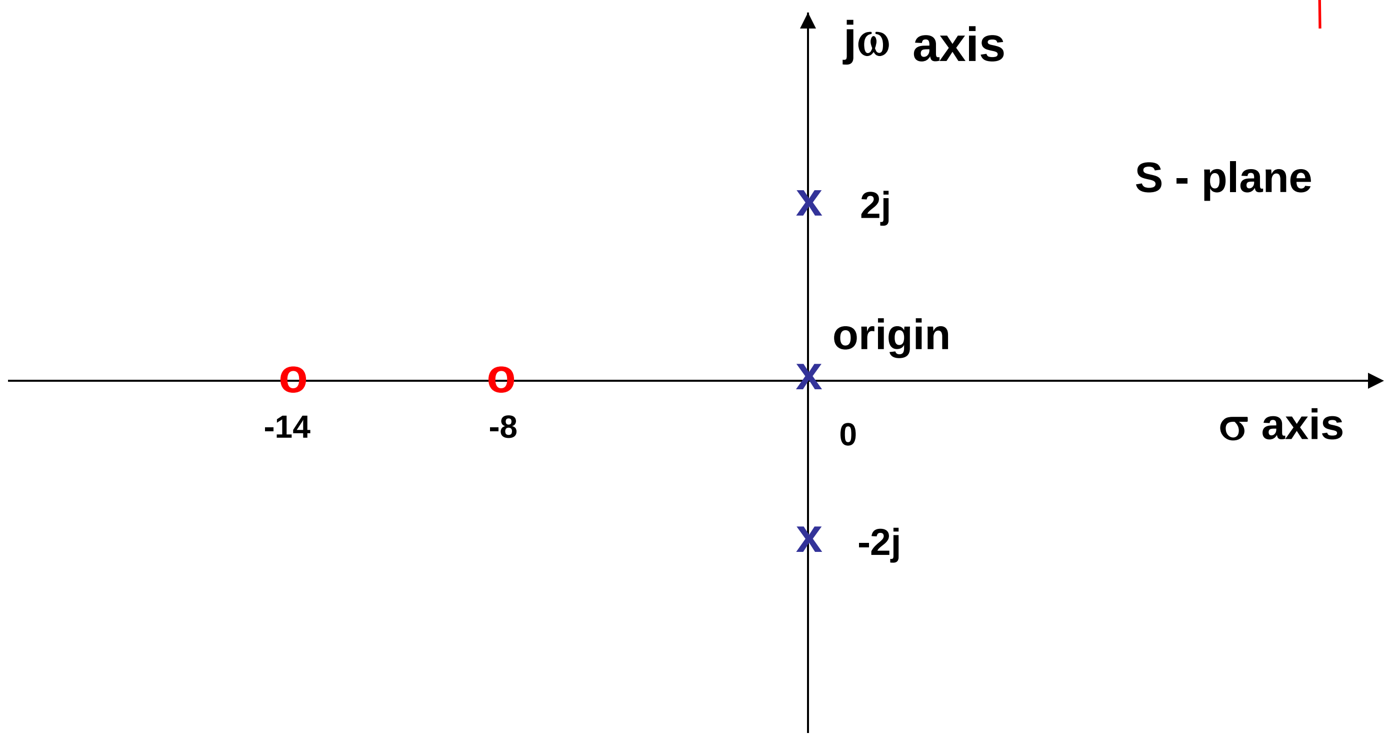


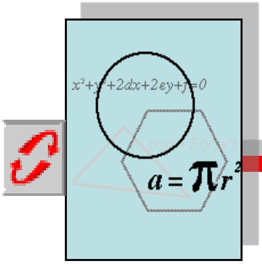
Given the transfer function. Plot the poles and zeros in the s-plane.

$$H(s) = \frac{(s+8)(s+14)}{s(s^2+4)}$$

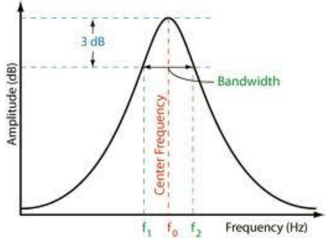


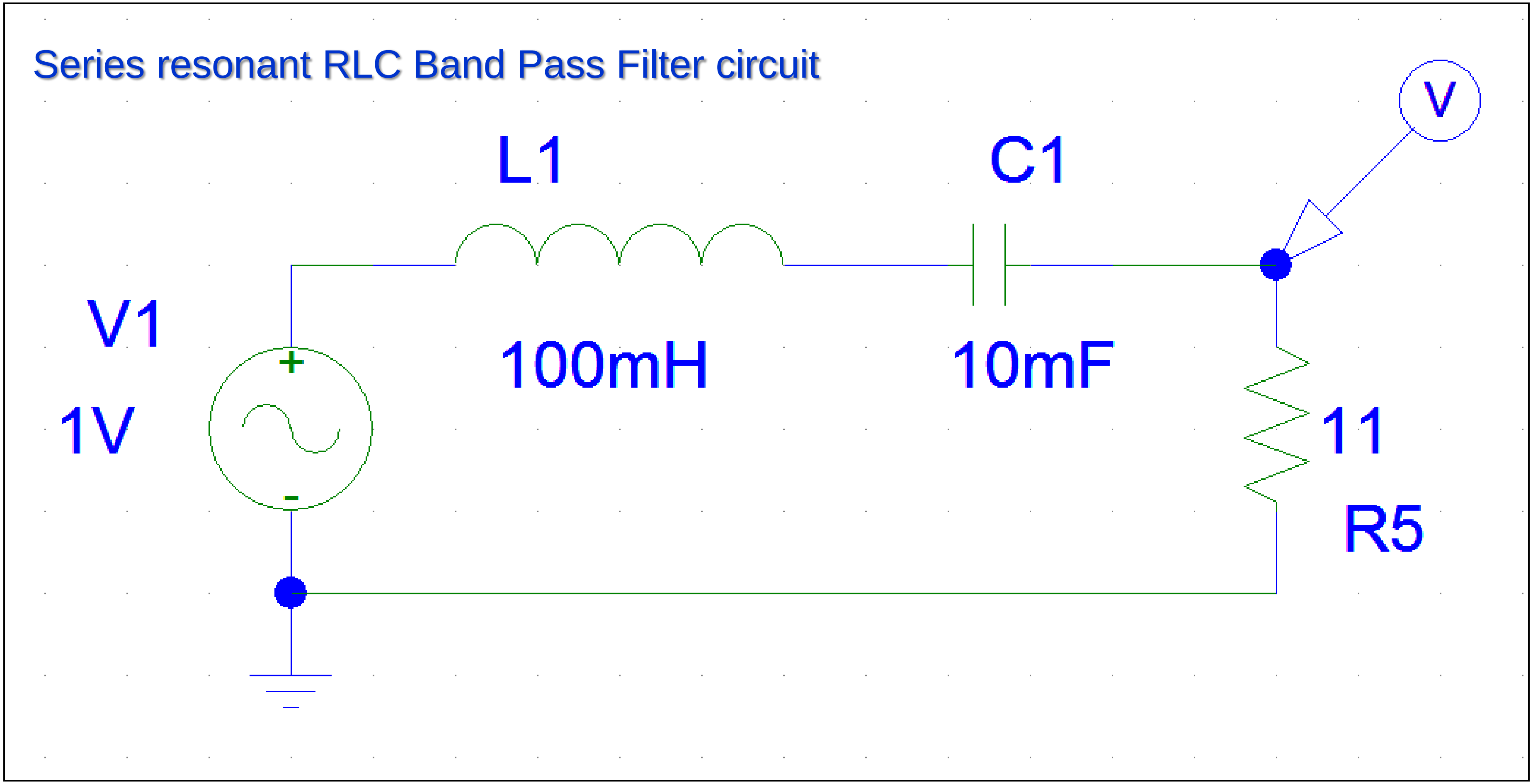
S-plane



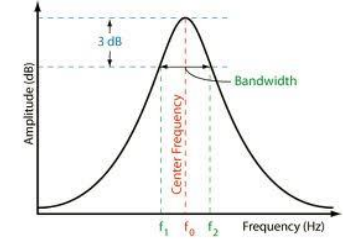


FILTERS – Transfer Functions/Bode Plots





FILTERS – Transfer Functions/Bode Plots



Impedance circuit diagram

jX_L $\qquad$ $-jX_C$

$j\omega L = sL$ $\qquad$ $-j\dfrac{1}{\omega C} = \dfrac{1}{sC}$

Z_2 $\qquad$ Z_3

Z_T $\qquad$ R $\qquad$ Z_1

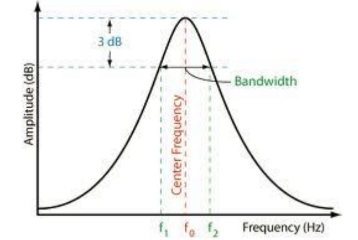
$$Z_T = Z_1 + Z_2 + Z_3$$

$$Z_T(\omega) = R + j\omega L + \frac{1}{j\omega C}$$

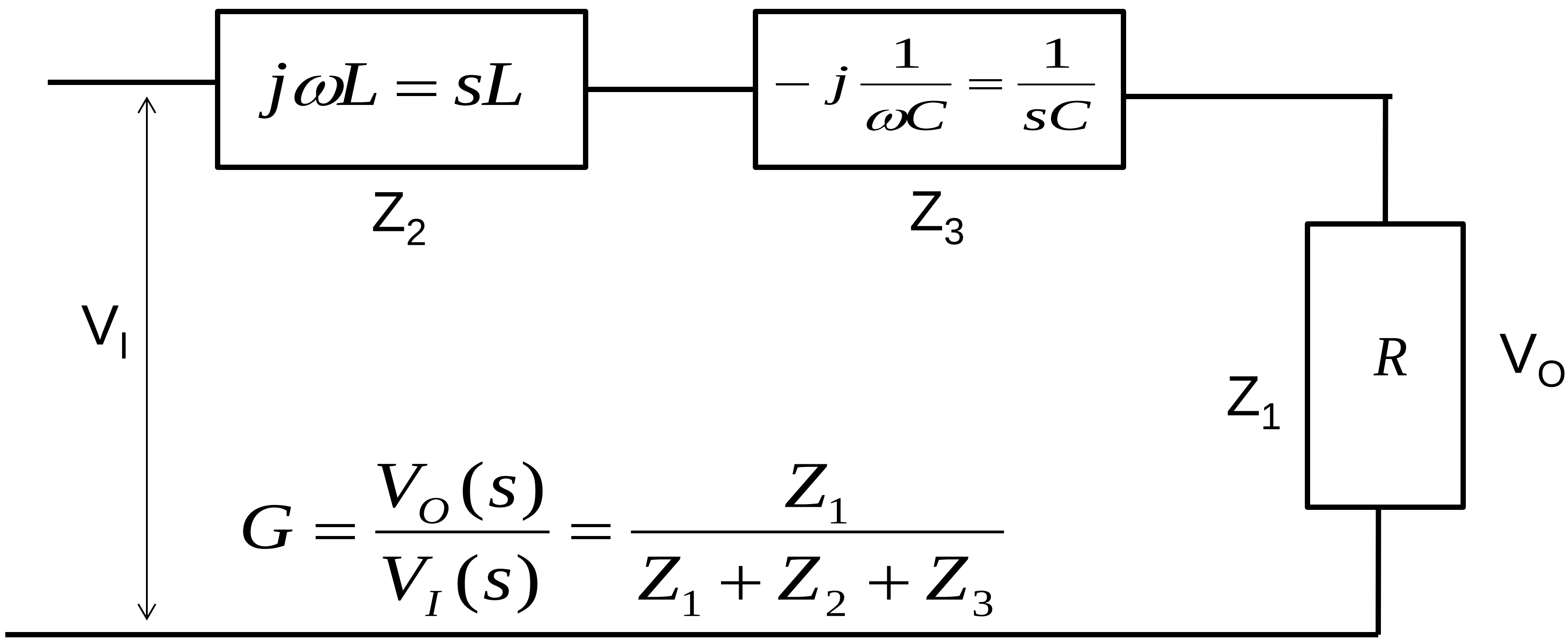
$\boxed{let\ \ j\omega = s}$

$$Z_T(s) = R + sL + \frac{1}{sC}$$

FILTERS – Transfer Functions/Bode Plots



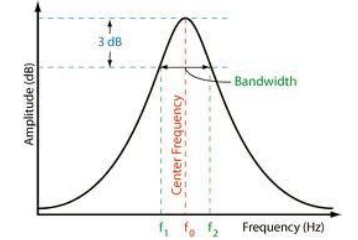
Transfer Function H(s)



$$G = \frac{V_O(s)}{V_I(s)} = \frac{Z_1}{Z_1 + Z_2 + Z_3}$$

$$H(s) = \frac{V_O(s)}{V_I(s)} = \frac{R}{R + sL + \frac{1}{sC}}$$

FILTERS – Transfer Functions/Bode Plots



$$H(s)=\frac{R}{R+sL+\frac{1}{sC}}=\frac{R}{\frac{sRC+s^2LC+1}{sC}}=\frac{sRC}{sRC+s^2LC+1}$$

$$H(s)=\frac{sRC}{LC\left(\frac{sR}{L}+s^2+\frac{1}{LC}\right)}=\frac{s\frac{R}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

Quality factor

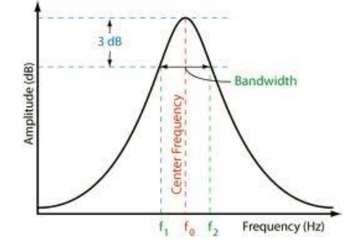
$$Q_o=\frac{I^2X_L}{I^2R}=\frac{X_L}{R}=\frac{\omega_oL}{R}\quad\Rightarrow\quad\boxed{\frac{R}{L}=\frac{\omega_o}{Q_o}}$$

Resonant frequency

$$f_o=\frac{1}{2\pi\sqrt{LC}}\quad\Rightarrow\quad\boxed{\frac{1}{LC}=\omega_o{}^2}$$

$$H(j\omega)=\frac{s\frac{\omega_o}{Q_o}}{s^2+\frac{\omega_o}{Q_o}s+\omega_o{}^2}$$

FILTERS – Transfer Functions/Bode Plots



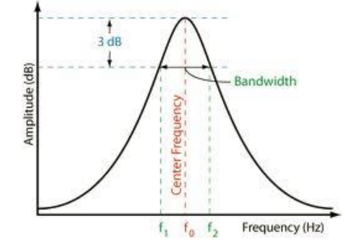
$$H(s)=\frac{s\frac{R}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

L = 100 mH, C = 10 mF, R = 11 Ω

$$H(s)=\frac{110s}{s^2+110s+1000}=\frac{110s}{(s+10)(s+100)}$$

$$H(s)=\frac{110s}{1000\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}=\frac{0.11s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$$

FILTERS – Transfer Functions/Bode Plots



Pzapplet Run: L = 100 mH, C = 10 mF, R = 11 Ω

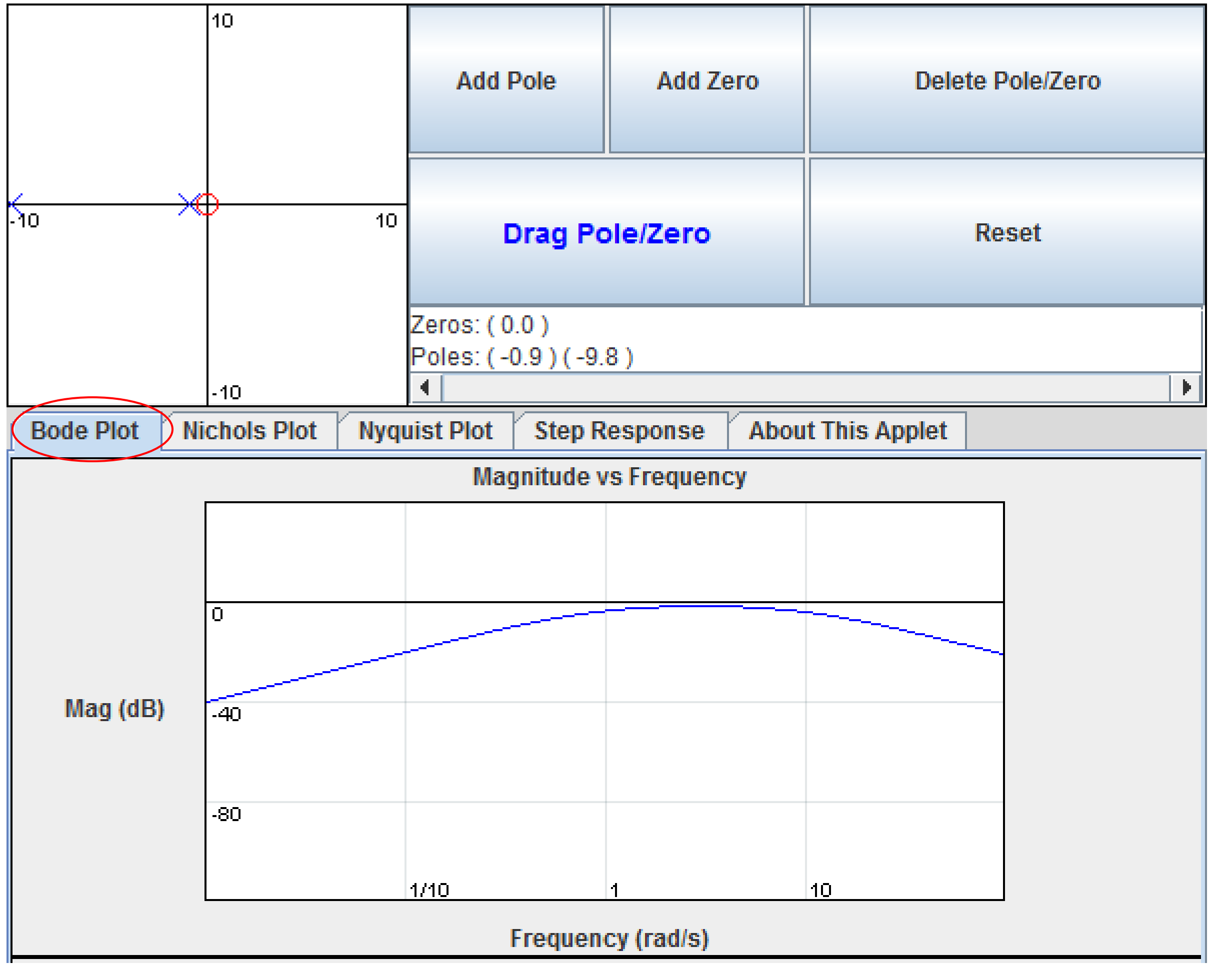
http://home.dei.polimi.it/guariso/pzapplet/index.html

Pzapplet (MIT)

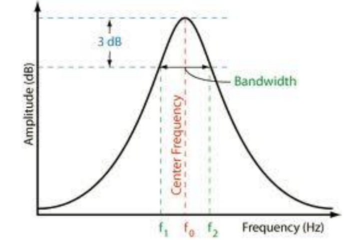
$$H(s) = \frac{0.11s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$$

Normalize

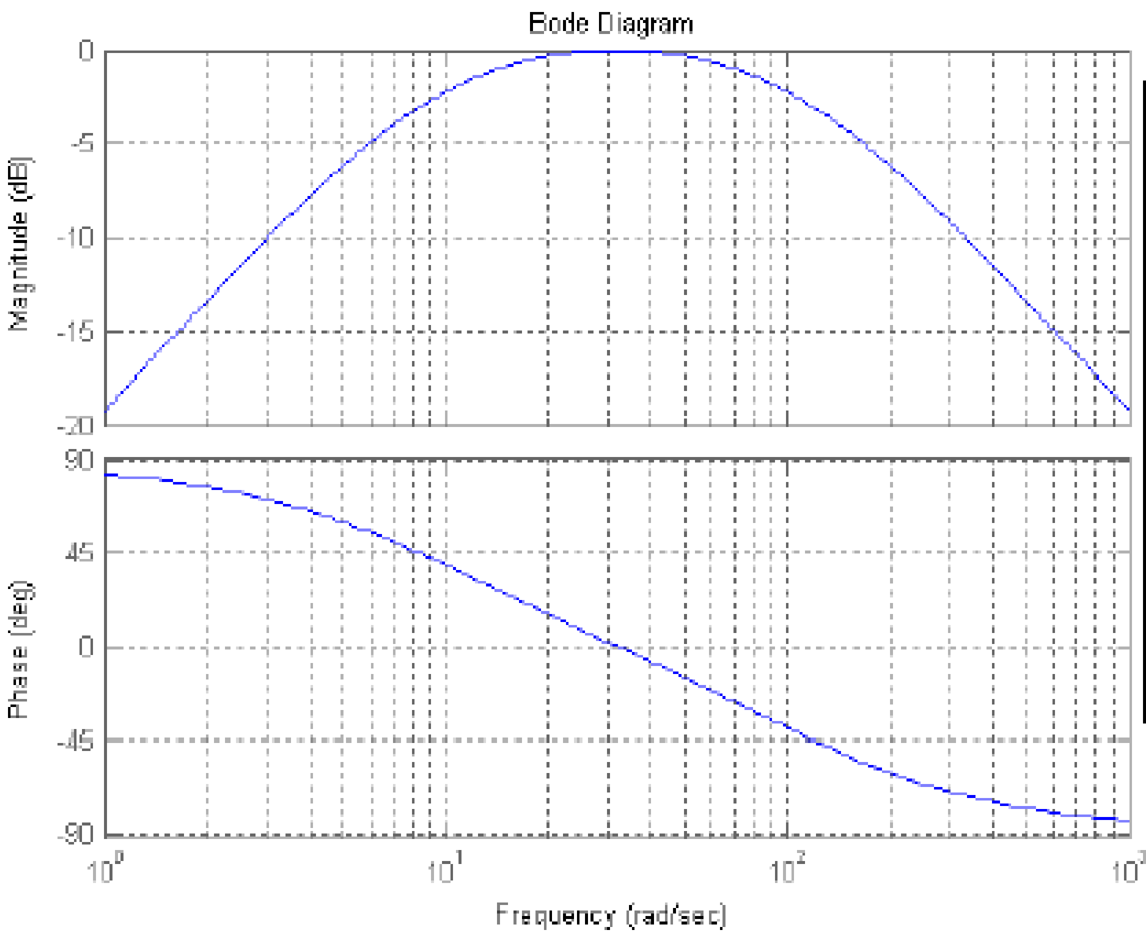
$$H(s) = \frac{0.011s}{\left(1+\frac{s}{1}\right)\left(1+\frac{s}{10}\right)}$$



FILTERS – Transfer Functions/Bode Plots



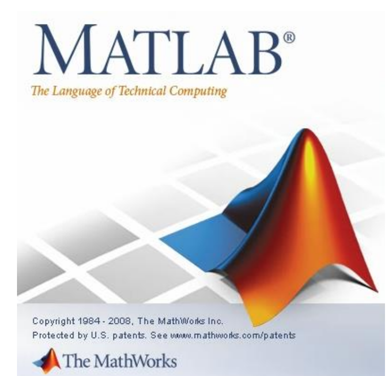
Matlab Run: L = 100 mH, C = 10 mF, R = 11 Ω



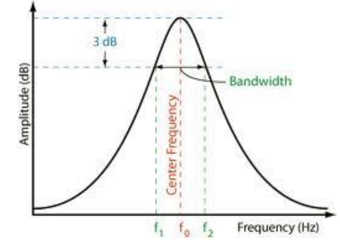

```
>> dn=[1 110 1000];
>> n=[0 110 0];
>> g=tf(n,dn)

Transfer function:
      110 s
------------------
s^2 + 110 s + 1000


>> bode (g)
>> grid on
```



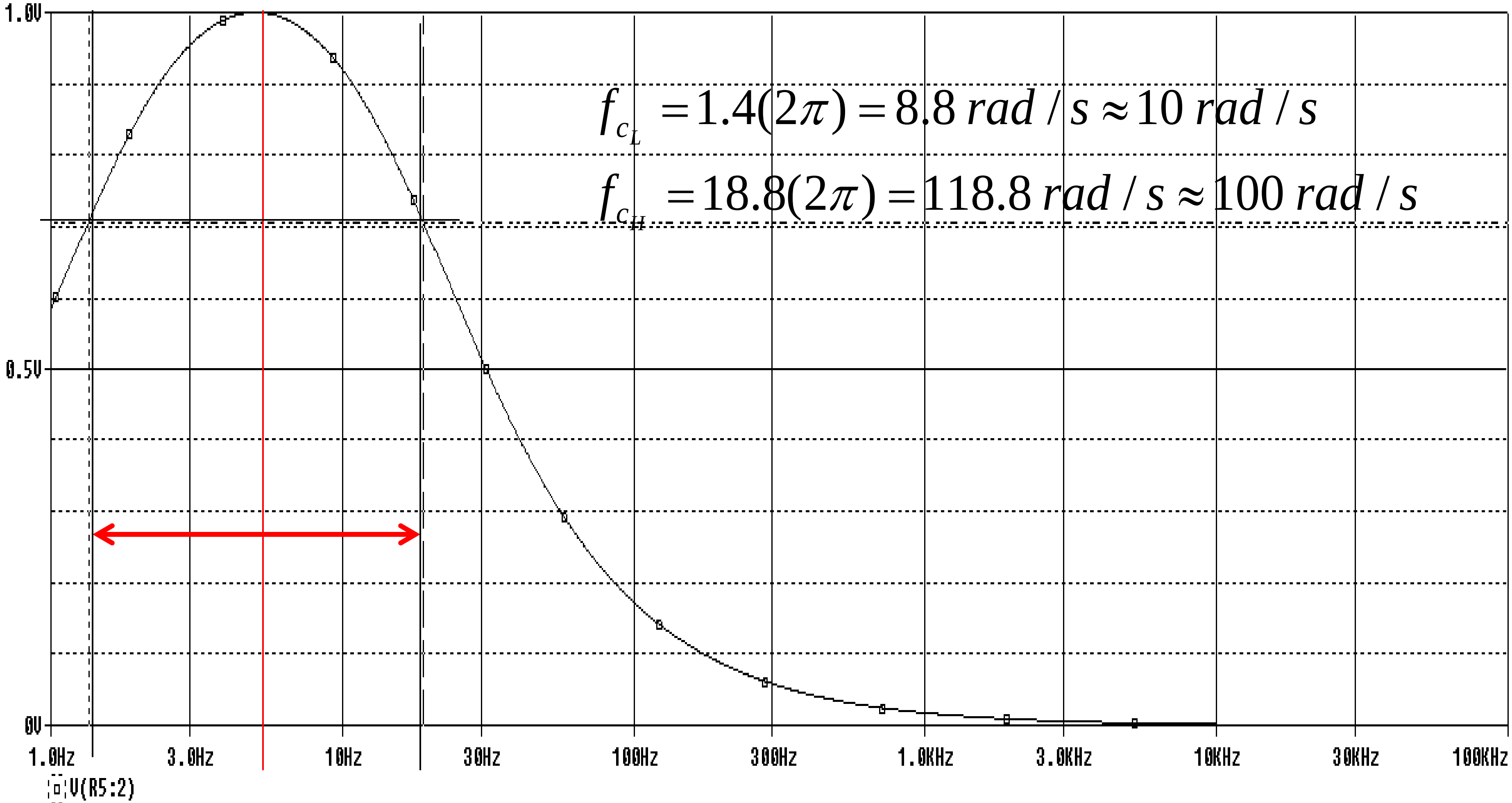
FILTERS – Transfer Functions/Bode Plots



Pspice Run: L = 100 mH, C = 10 mF, R = 11 Ω

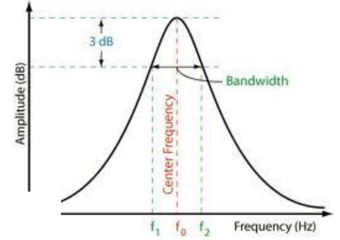
$$f_{c_L} = 1.4(2\pi) = 8.8\ rad/s \approx 10\ rad/s$$

$$f_{c_H} = 18.8(2\pi) = 118.8\ rad/s \approx 100\ rad/s$$

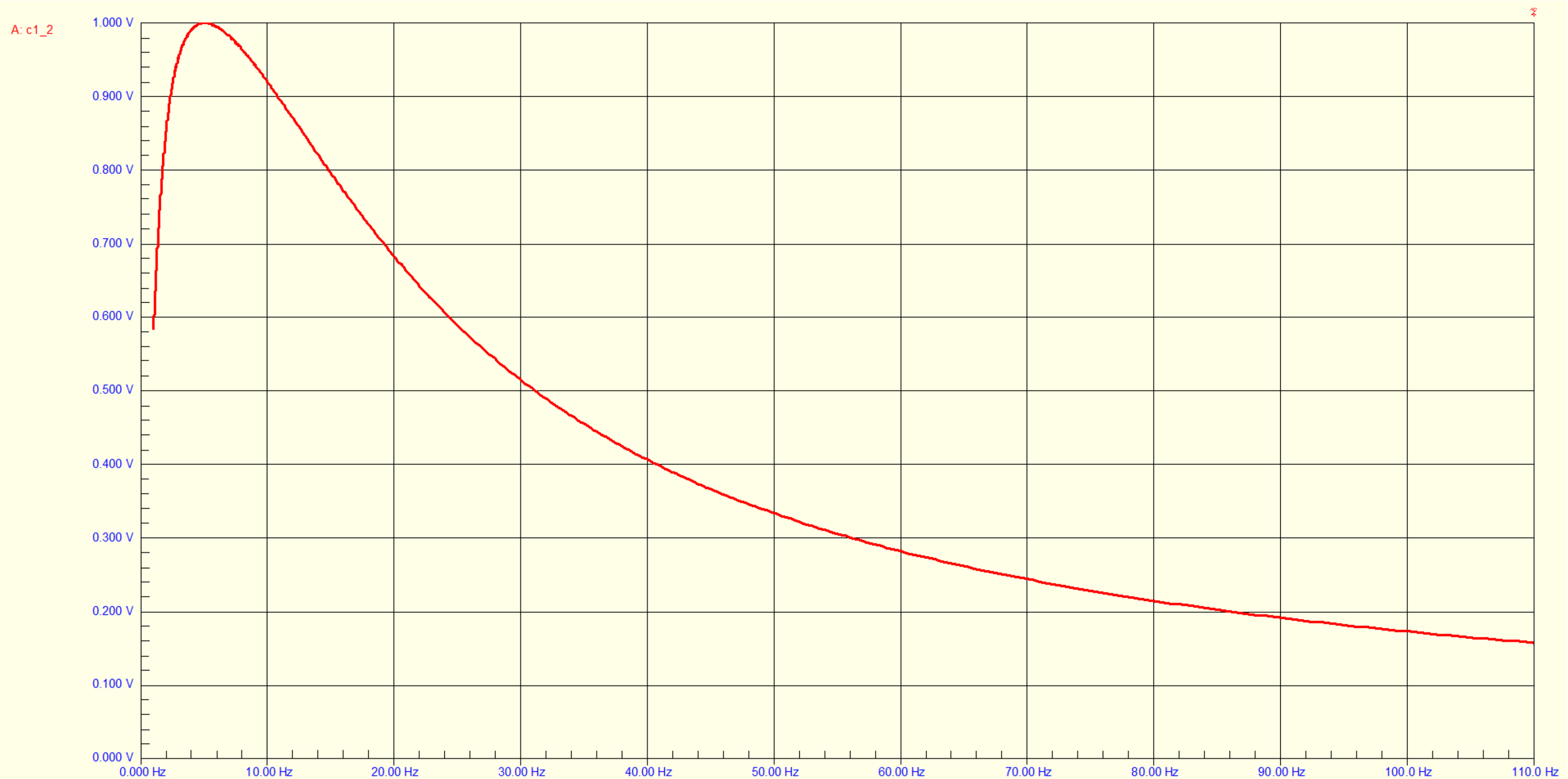


1.4 **18.8**

FILTERS – Transfer Functions/Bode Plots



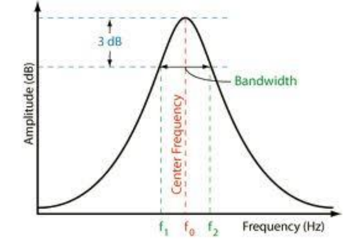
Circuit Maker Run: L = 100 mH, C = 10 mF, R = 11 Ω



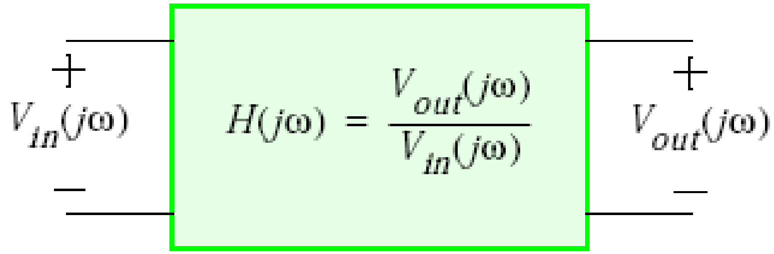
Measurement Cursors

1	c1_2	X: 110.00	Y: 157.50m
2	c1_2	X: 110.00	Y: 157.50m
Minimum 1 . . 2		Min: 157.50m	

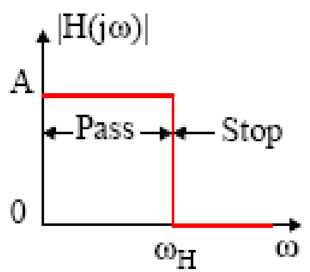
FILTERS



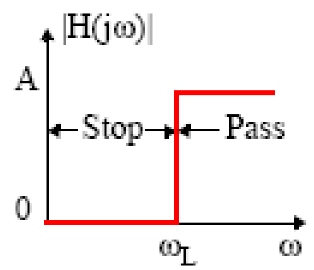
Ideal Filters



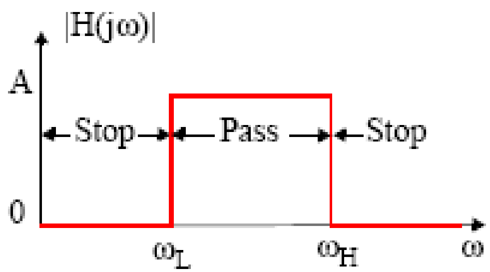
Low Pass Filter



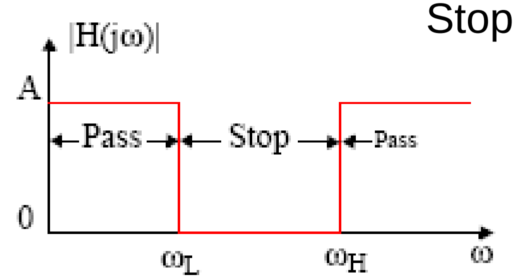
High Pass Filter



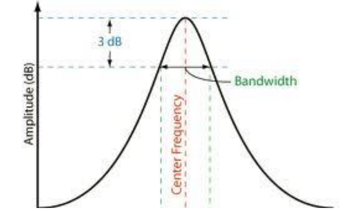
Band Pass Filter



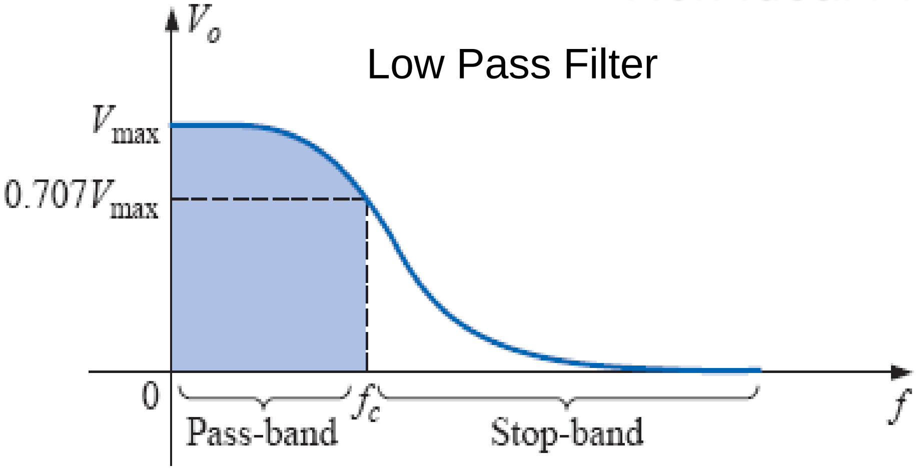
Stop Band Filter

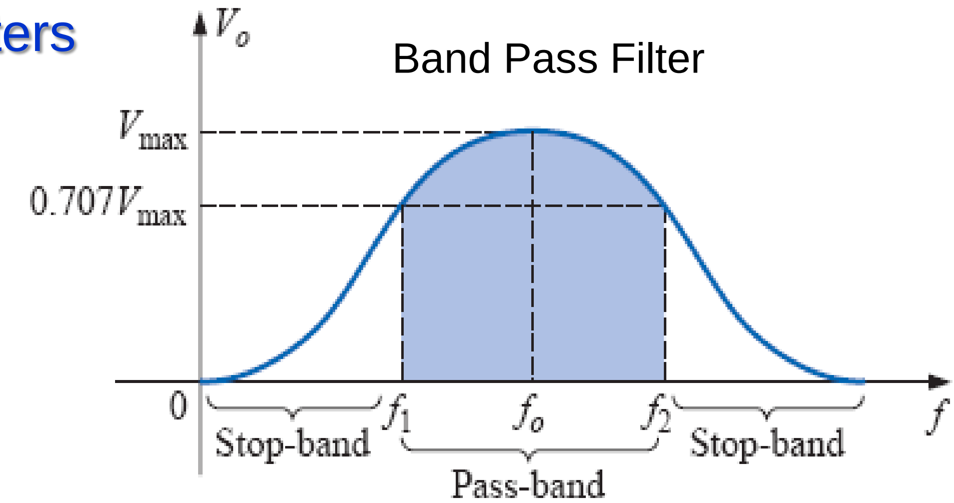


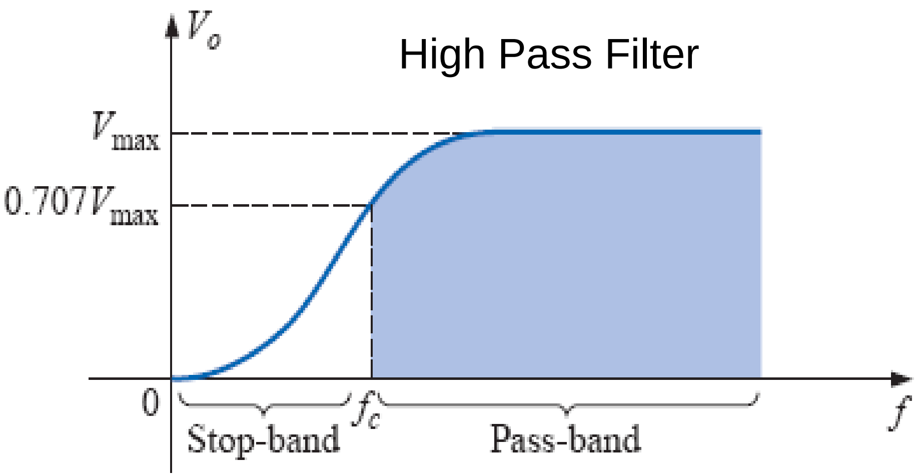
FILTERS

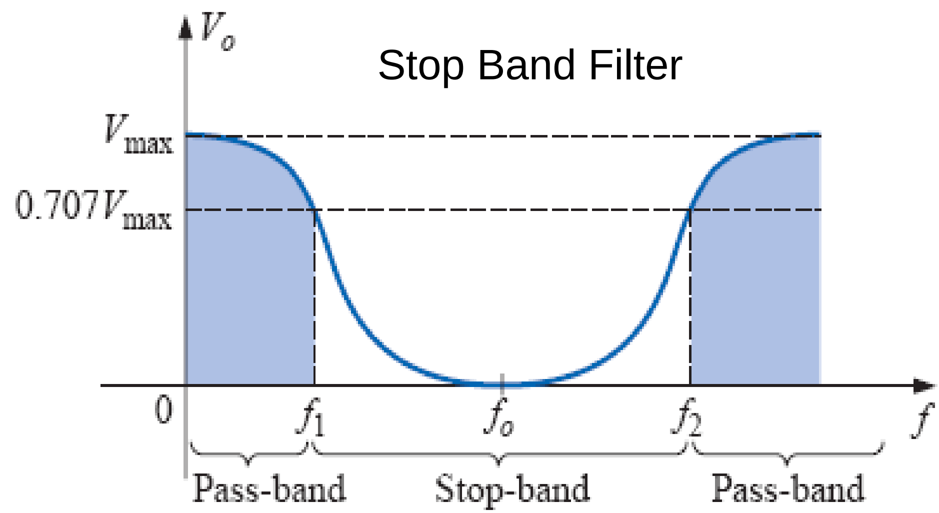


Non Ideal Filters

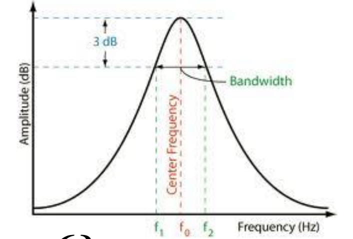




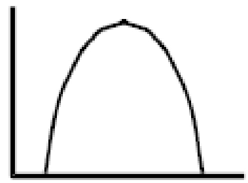


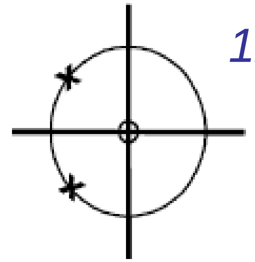


FILTERS – Transfer Functions/Bode Plots



BANDPASS

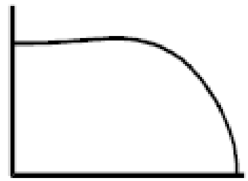


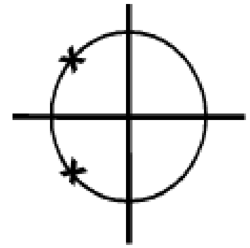


$1z_{Real}$

$$H(s)=\frac{s\,\dfrac{\omega_o}{Q_o}}{s^2+\dfrac{\omega_o}{Q_o}s+\omega_o{}^2}$$

LOWPASS

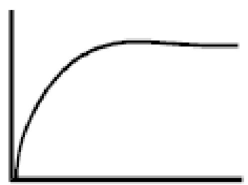


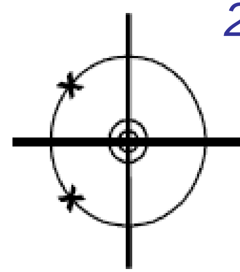


$0z$

$$H(s)=\frac{\omega_o{}^2}{s^2+\dfrac{\omega_o}{Q_o}s+\omega_o{}^2}$$

HIGHPASS

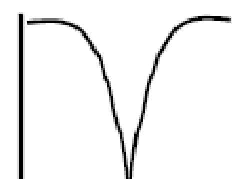


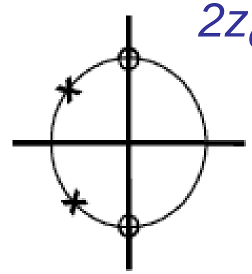


$2z_{Real}$

$$H(s)=\frac{s^2}{s^2+\dfrac{\omega_o}{Q_o}s+\omega_o{}^2}$$

BAND
REJECT

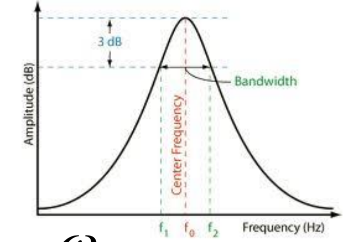




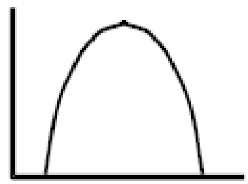
$2z_{Complex}$

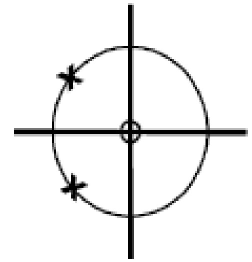
$$H(s)=\frac{s^2+\omega_z{}^2}{s^2+\dfrac{\omega_o}{Q_o}s+\omega_o{}^2}$$

FILTERS – Transfer Functions/Bode Plots



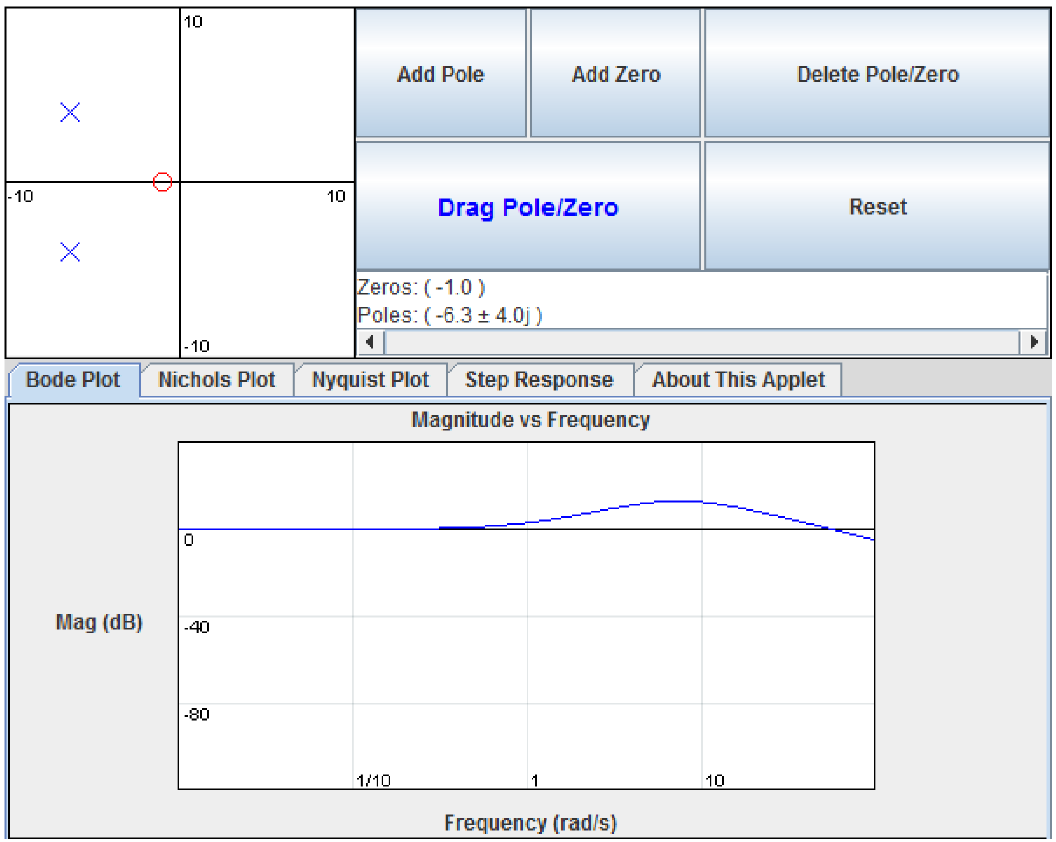
BANDPASS



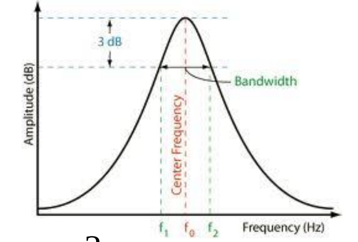


$$H(s) = \frac{s \dfrac{\omega_o}{Q_o}}{s^2 + \dfrac{\omega_o}{Q_o} s + \omega_o{}^2}$$

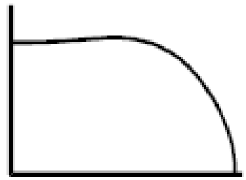
MIT
PoleZero Applet

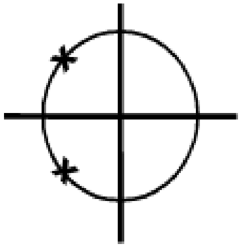


FILTERS – Transfer Functions/Bode Plots



LOWPASS

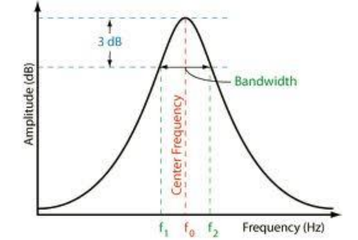




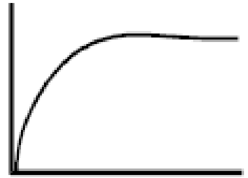
$$H(s) = \frac{\omega_o^{\ 2}}{s^2 + \frac{\omega_o}{Q_o} s + \omega_o^{\ 2}}$$

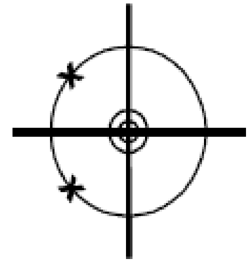
MIT
PoleZero Applet

FILTERS – Transfer Functions/Bode Plots



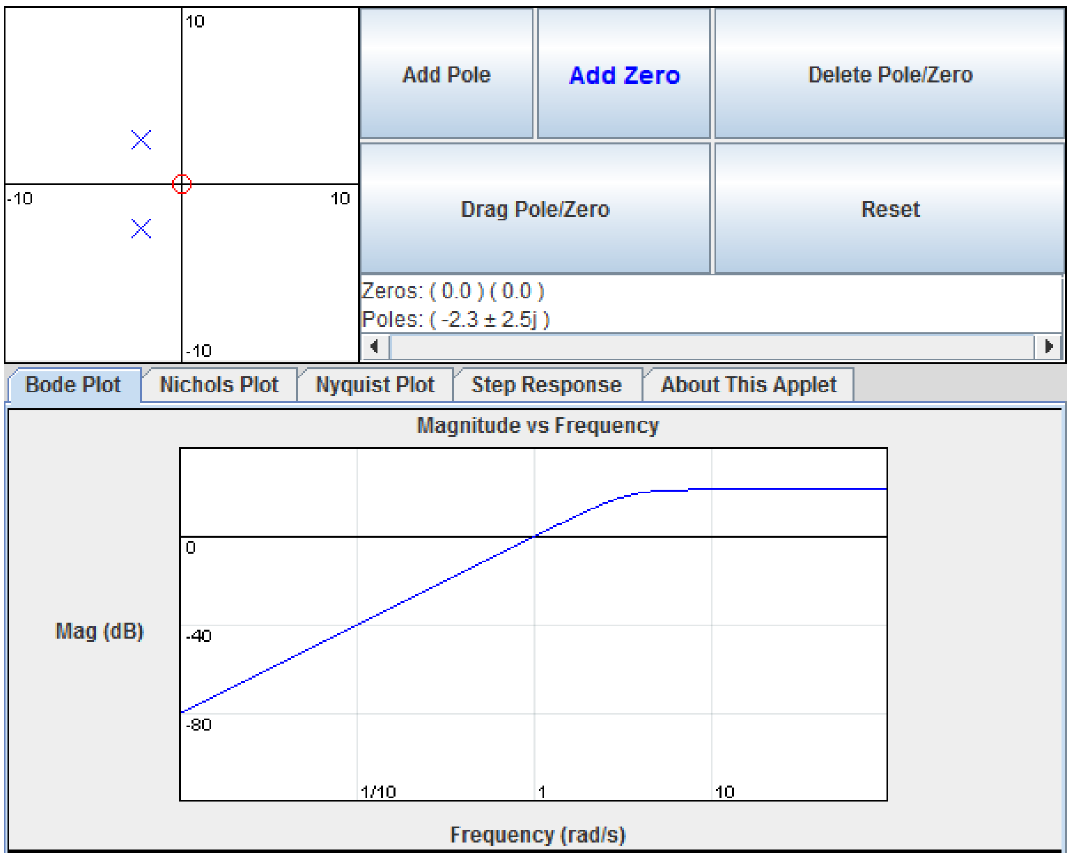
HIGHPASS



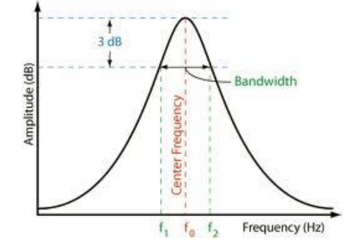


$$H(s) = \frac{s^2}{s^2 + \frac{\omega_o}{Q_o} s + \omega_o^2}$$

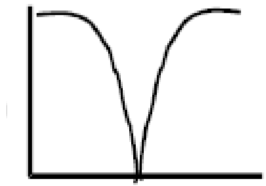
MIT
PoleZero Applet

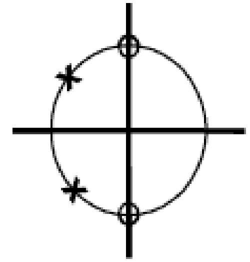


FILTERS – Transfer Functions/Bode Plots



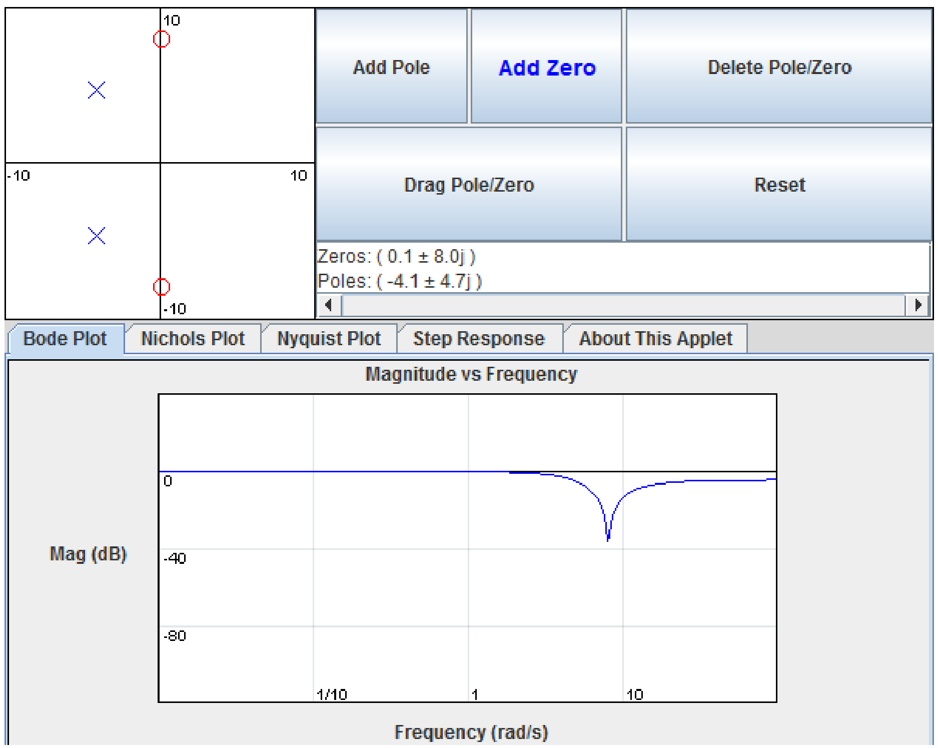
BAND
REJECT



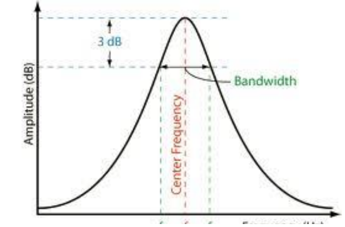


$$H(s) = \frac{s^2 + \omega_z^{\,2}}{s^2 + \frac{\omega_o}{Q_o} s + \omega_o^{\,2}}$$

MIT
PoleZero Applet



BODE PLOT - LOGARITHMS



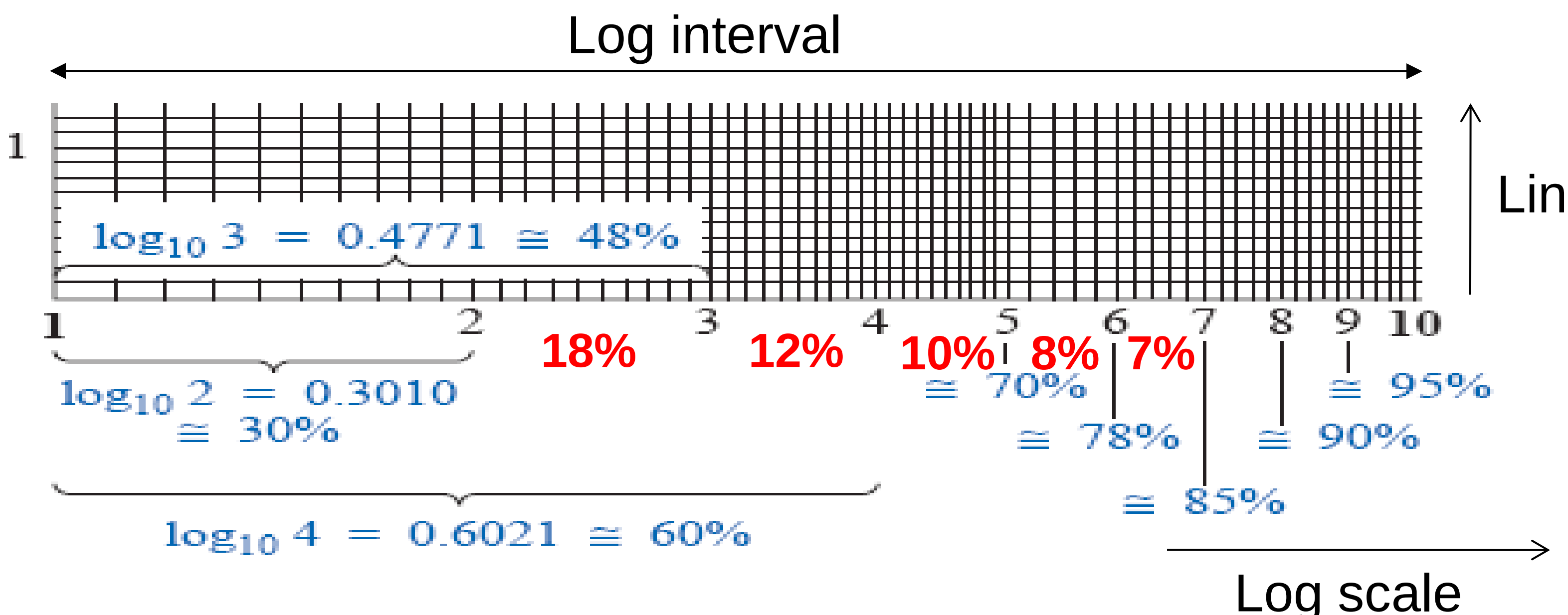
$$y = 10^x \quad \leftrightarrow \quad x = \log_{10} y$$

Such that:

$$\log_{10} 1 = 0$$
$$\log_{10} 10 = 1$$
$$\log_{10} 100 = 2$$
$$\log_{10} 1000 = 3$$

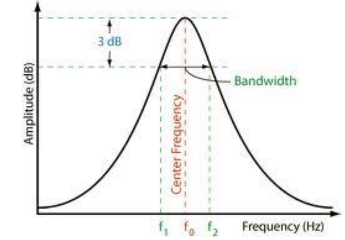
Graph paper is available in the *semilog* and *log-log varieties*.

The spacing of the log scale is determined by taking the common log of the number. The scaling starts with 1, since log 1 = 0.
The distance between 1 and 2 is determined by
log 2 = 0.3010, (30% of the full distance of a log interval).

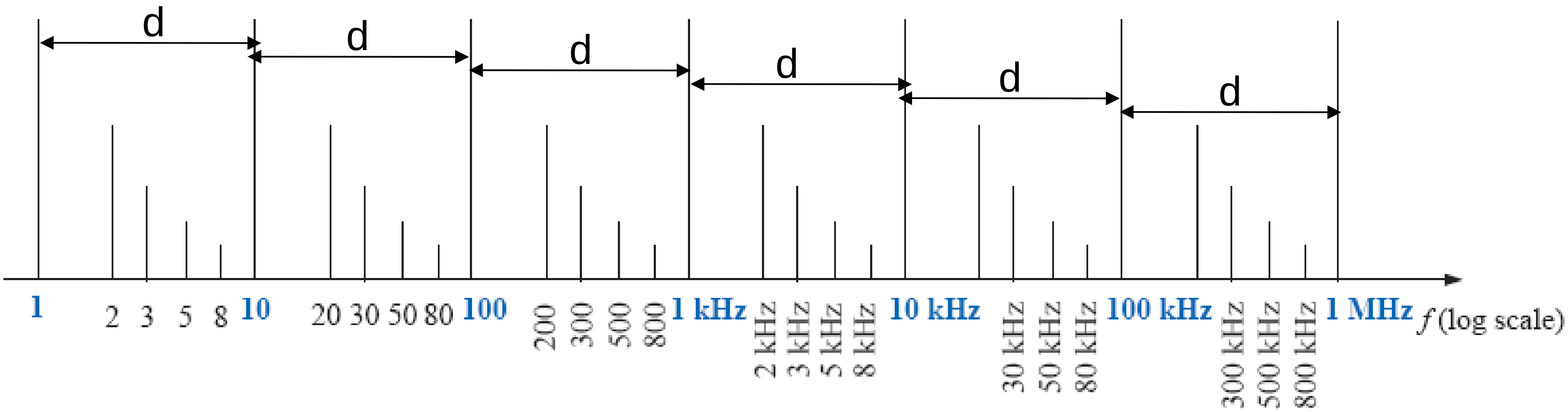


spacing = log N

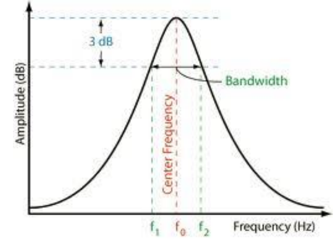
FILTERS

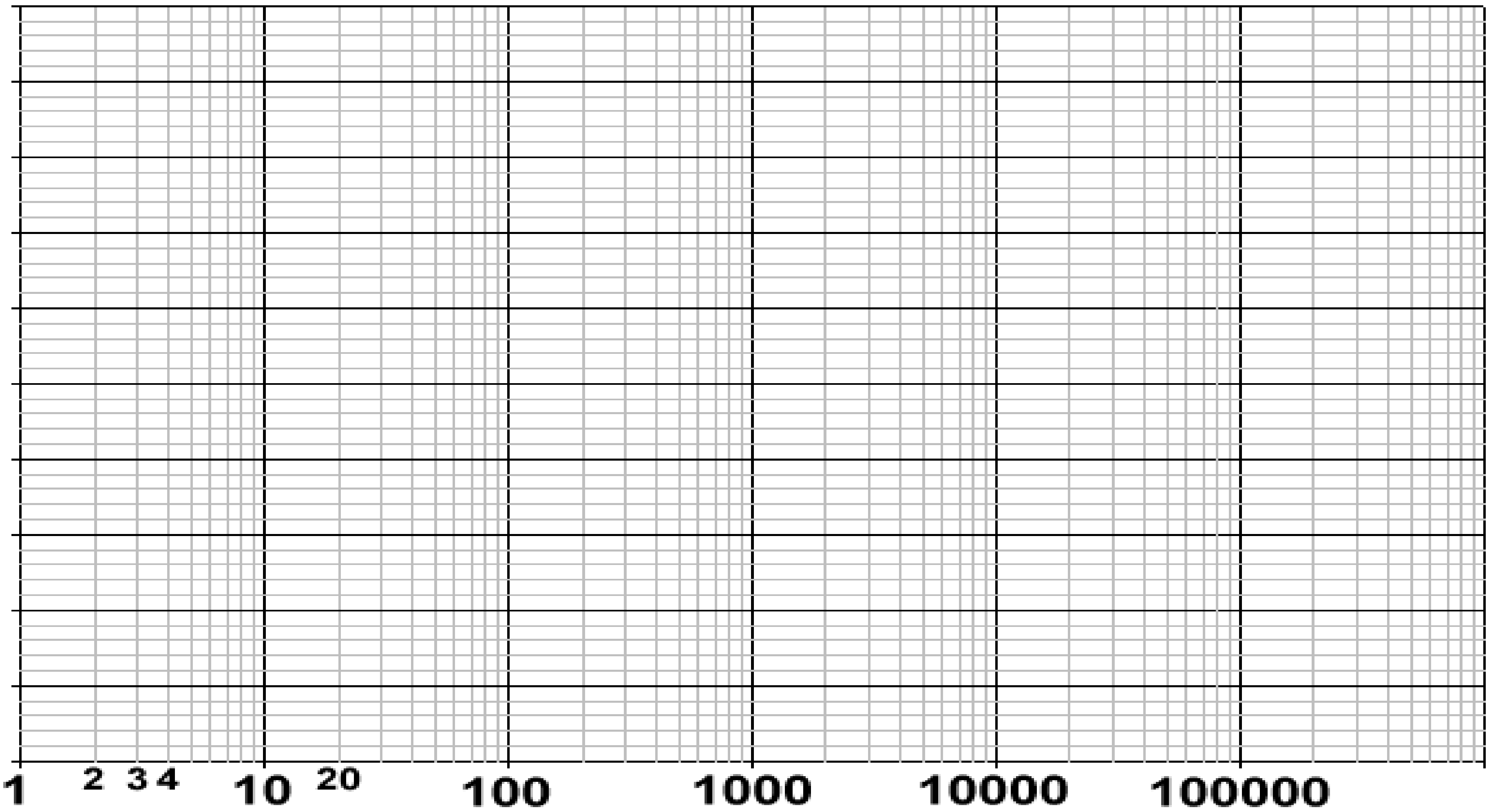


Frequency log scale

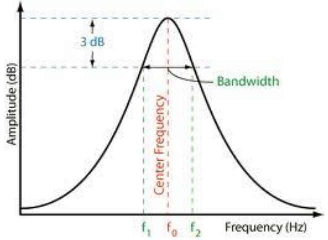


Semilog Graph





FILTERS



$$y = 10^x \quad \leftrightarrow \quad x = \log_{10} y$$

Properties of logarithms

$$\log_{10} 1 = 0$$

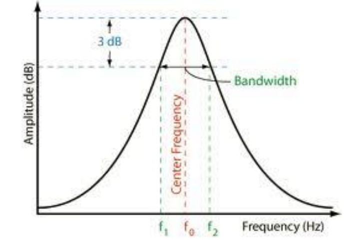
$$\log_{10} ab = \log_{10} a + \log_{10} b$$

$$\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b$$

$$\log_{10} a^n = n \log_{10} a$$

$$\log_{10} \frac{1}{b} = \log_{10} b^{-1} = -\log_{10} b$$

DECIBELS



Power Gain:

Two levels of power can be compared using a unit of measure called the *bel:*

$$\boxed{B = \log_{10} \frac{P_2}{P_1}} \qquad \text{(bels)}$$

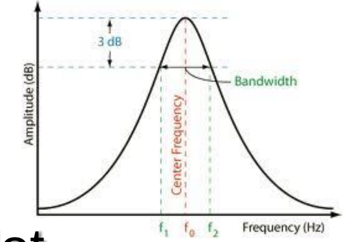
A decibel is defined as:

$$\boxed{dB = 10\log_{10} \frac{P_2}{P_1} = 20\log_{10} \frac{V_2}{V_1}} \qquad \text{(decibel - dB)}$$

A decibel referenced to 1 mW is given as:

$$\boxed{dB_m = 10\log_{10} \left.\frac{P}{1\,mW}\right|_{600\Omega}}$$

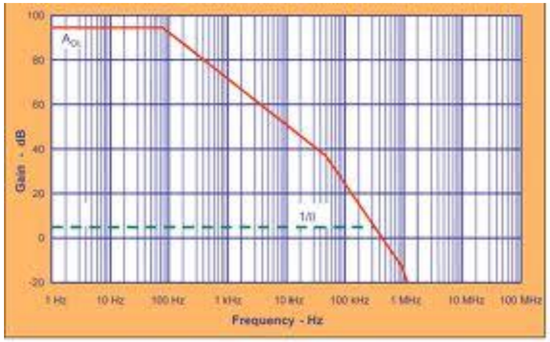
FILTERS – Transfer Functions/Bode Plots



Given a transfer function we can generate a BODE plot.

A **Bode plot** is a graph of the transfer function H(jω) versus frequency, ω, plotted with a log-frequency axis, to show the system's frequency response.

A_{dB}



ω

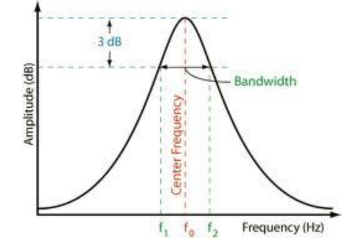
$$H(s)=\frac{k(s+z)}{s\,(s+p)} \qquad z,p,k\in R$$

$$H(jw)=\frac{k(j\omega+z)}{(j\omega)(j\omega+p)}=\frac{kz\left(1+\frac{j\omega}{z}\right)}{(j\omega)p\left(1+\frac{j\omega}{p}\right)}=\frac{K\left(1+\frac{j\omega}{z}\right)}{(j\omega)\left(1+\frac{j\omega}{p}\right)}$$

$$A_{dB=}20\log\left|H(j\omega)\right|$$

$$A_{dB}=20\log\left|K\right|+20\log\left|\left(1+\frac{j\omega}{z}\right)\right|-20\log\left|j\omega\right|-20\log\left|\left(1+\frac{j\omega}{p}\right)\right|$$

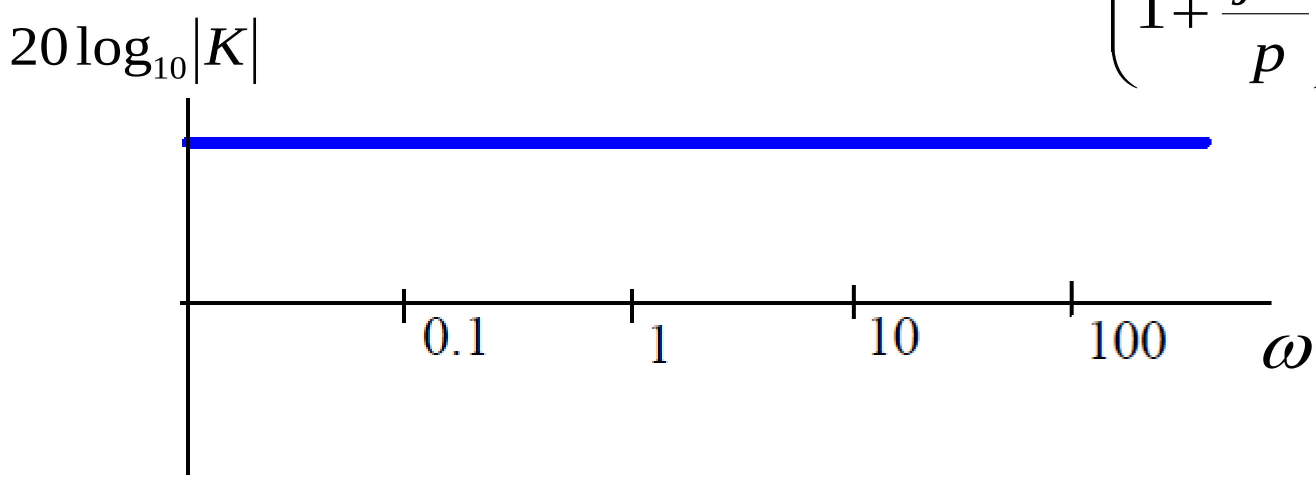
FILTERS – Transfer Functions/Bode Plots

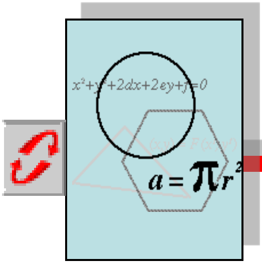


Effect of constant terms:

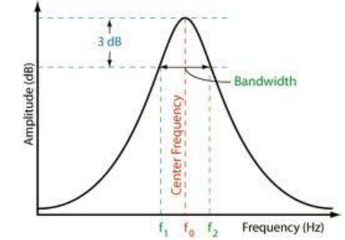
Constant terms such as ***K*** contribute a straight horizontal line of magnitude 20 $\log_{10}(K)$.

$$H(jw) = \frac{K(j\omega)\left(1+\dfrac{j\omega}{z}\right)}{\left(1+\dfrac{j\omega}{p}\right)}$$





FILTERS – Transfer Functions/Bode Plots



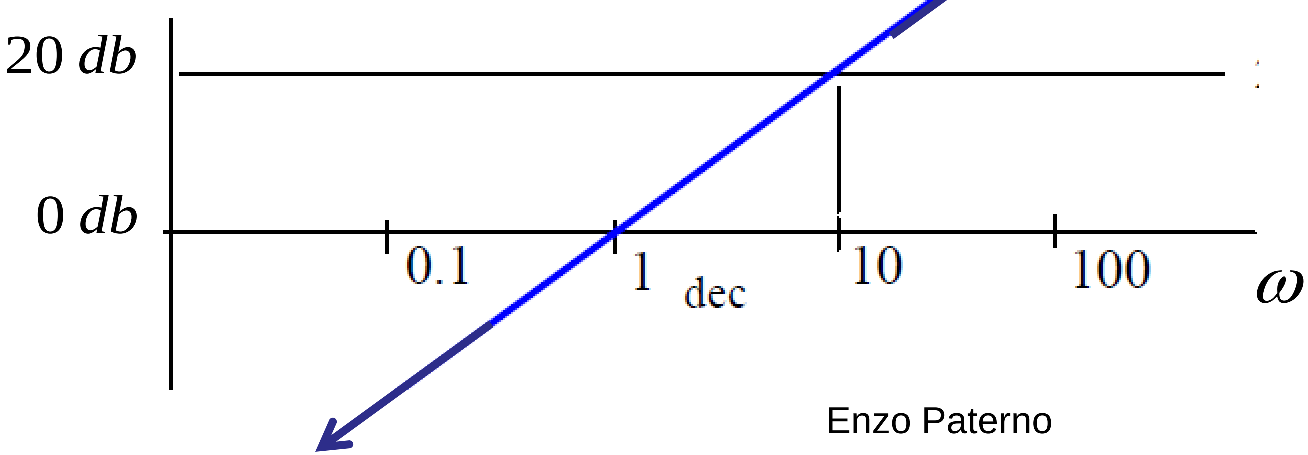
Effect of a ZERO at the origin:

A **zero** at the origin occurs when ***jω*** multiplies the numerator. Each occurrence of this causes a positively sloped line passing through *ω* = 1 with a rise of 20 db over a decade continuously.

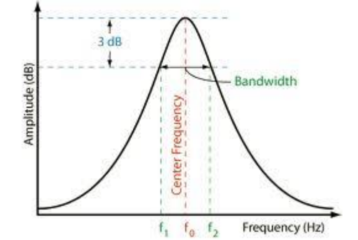
$$H(jw) = \frac{K(j\omega)\left(1+\frac{j\omega}{z}\right)}{\left(1+\frac{j\omega}{p}\right)}$$

$$20\log_{10}|j\omega| \to 0dB @ \omega = 1$$

$$20\log_{10}|j\omega| \to 20dB @ \omega = 10$$



FILTERS – Transfer Functions/Bode Plots



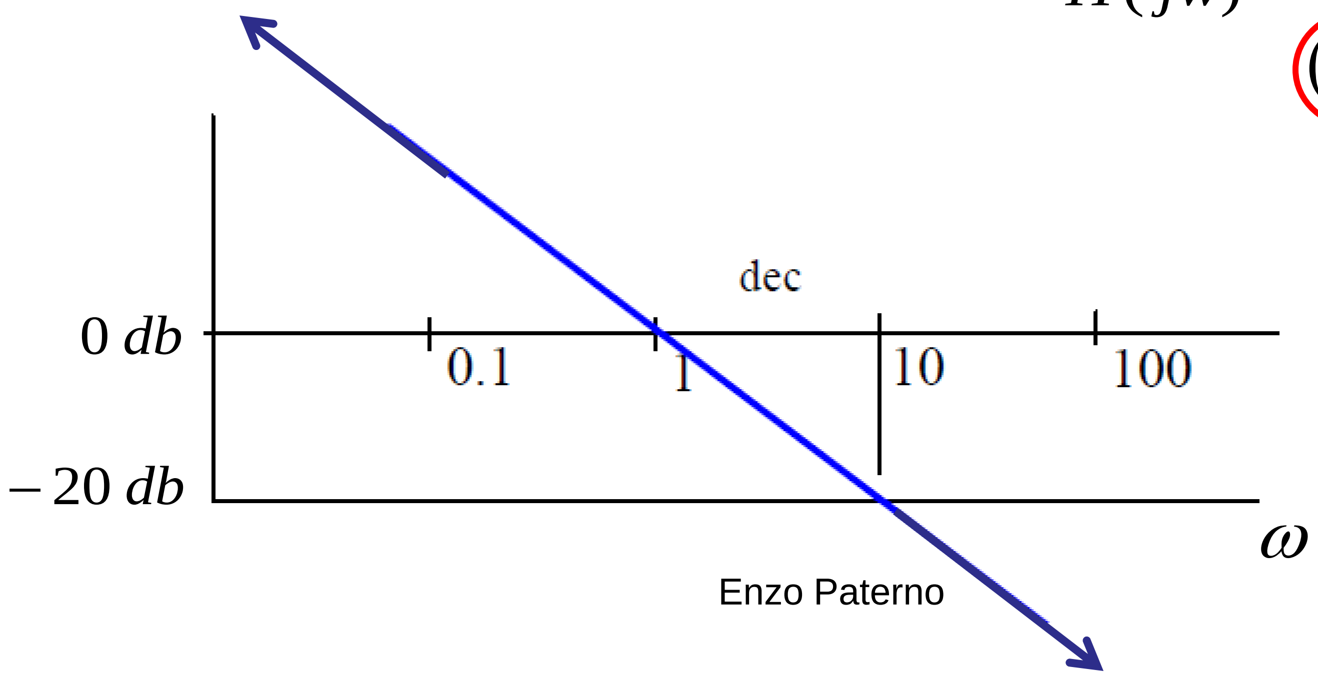
Effect of a POLE at the origin:

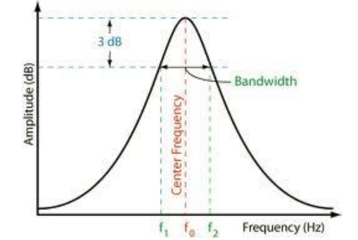
A **pole** at the origin occurs when ***jω*** multiplies the denominator. Each occurrence of this causes a negatively sloped line passing through $\omega = 1$ with a drop of 20 db over a decade continuously.

$$-20\log_{10}|j\omega| \rightarrow 0dB\,@\,\omega = 1$$

$$-20\log_{10}|j\omega| \rightarrow -20dB\,@\,\omega = 10$$

$$H(jw) = \frac{K\left(1+\dfrac{j\omega}{z}\right)}{(j\omega)\left(1+\dfrac{j\omega}{p}\right)}$$





Effect of ZEROS not at the origin:

Zeros not at the origin are indicated by **(1+*j ω/z*)** terms. ***z*** in each of the terms is called the break frequency or the critical frequency. Below the break frequency they do not contribute to the log magnitude (0 *db*) and above the break frequency, they represent a ramp function of 20 db per decade with a positive slope continuously.

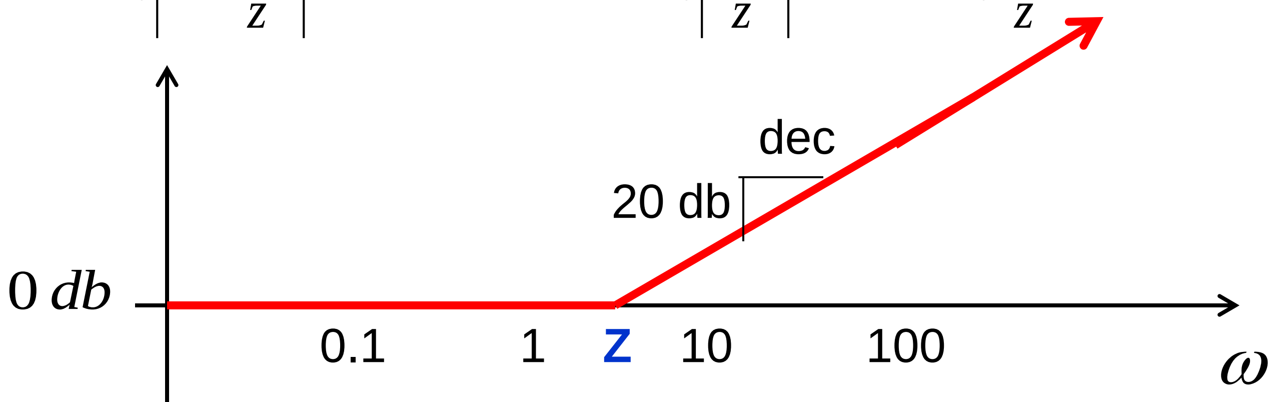
$$H(jw) = \frac{K\left(1+\dfrac{j\omega}{z}\right)}{(j\omega)\left(1+\dfrac{j\omega}{p}\right)}$$

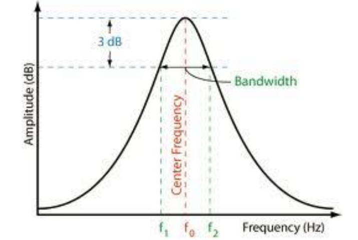
$$20\log_{10}\left|1+\frac{j\omega}{z}\right|; \quad \omega < z \rightarrow 0db$$

$$20\log_{10}\left|1+\frac{j\omega}{z}\right|; \quad \omega > z \rightarrow 20\log_{10}\left|\frac{j\omega}{z}\right| = 20\log_{10}\frac{\omega}{z}$$

$$20\log_{10}\left|1+j\frac{\omega}{z}\right|$$

$$20\log_{10}\sqrt{\left((1)^2+\left(\frac{\omega}{z}\right)^2\right)}$$





FILTERS – Transfer Functions/Bode Plots

Effect of POLES not at the origin:

Poles not at the origin are indicated by **(1+*j ω/p*)** terms. ***p*** in each of the terms is called the break frequency or the critical frequency. Below the break frequency they do not contribute to the log magnitude ($0\ db$) and above the break frequency, they represent a ramp function of 20 db per decade with a negative slope continuously.

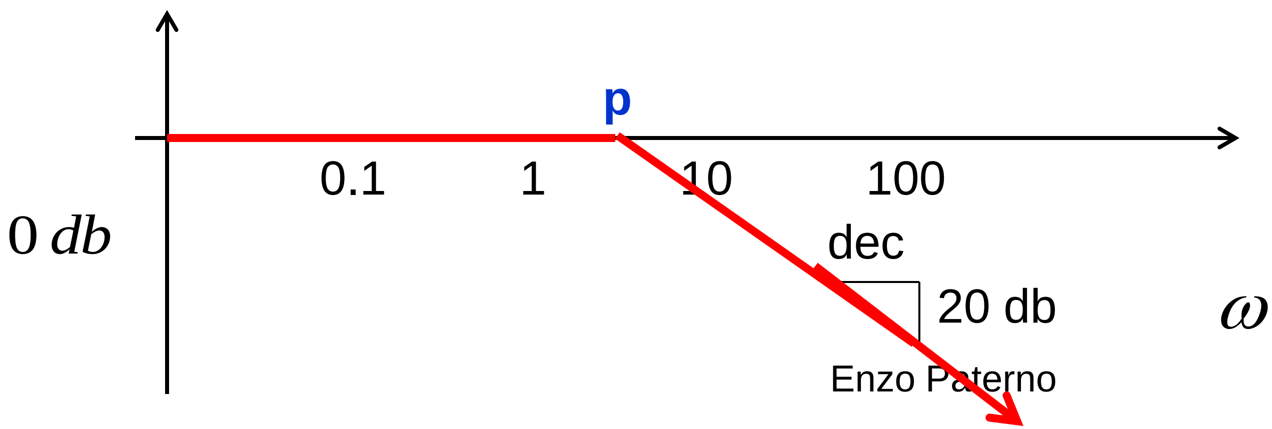
$$H(jw) = \frac{K\left(1+\dfrac{j\omega}{z}\right)}{(j\omega)\left(1+\dfrac{j\omega}{p}\right)}$$

$$-20\log_{10}\left|1+\frac{j\omega}{p}\right|; \quad \omega < p \rightarrow 0db$$

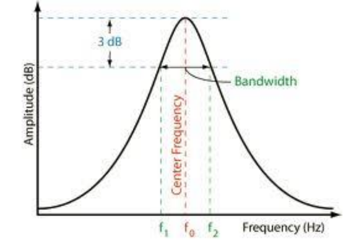
$$-20\log_{10}\left|1+\frac{j\omega}{p}\right|; \quad \omega > p \rightarrow -20\log_{10}\left|\frac{j\omega}{p}\right| = -20\log_{10}\frac{\omega}{p}$$

$$20\log_{10}\left|1+j\frac{\omega}{p}\right|$$

$$20\log_{10}\sqrt{\left((1)^2+\left(\frac{\omega}{p}\right)^2\right)}$$



FILTERS – Transfer Functions/Bode Plots

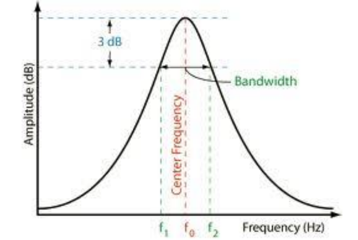


BODE Plot of a transfer function

To complete the log magnitude vs. frequency plot of a Bode diagram, we superposition all the lines of the different terms on the same plot.

Example: $$\left|H(s)\right| = \frac{1}{2s+100}$$

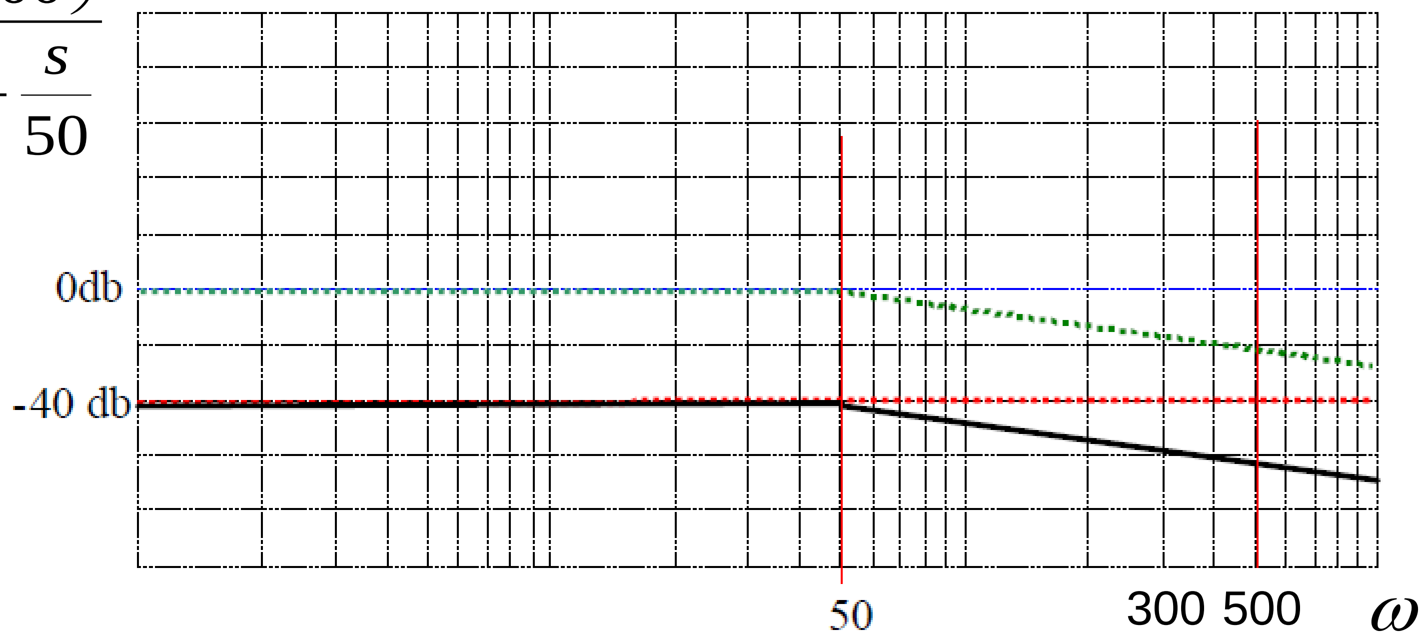
$$\left|H(s)\right| = \frac{1}{100\left(1+\frac{s}{50}\right)} = \frac{\left(\frac{1}{100}\right)}{1+\frac{s}{50}} \Rightarrow \frac{K}{1+\frac{s}{p}} \mapsto \begin{cases} K = 0.01 \\ p = 50 \end{cases}$$

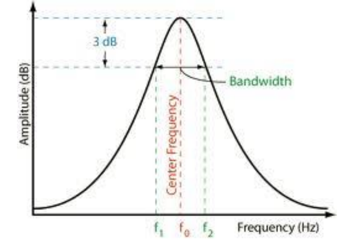


BODE Plot of a transfer function

$$|H(s)| = \frac{K}{1+\dfrac{s}{p}} \mapsto \begin{cases} K = 0.01 \\ p = 50 \end{cases}$$

$$20\log \frac{\left(\dfrac{1}{100}\right)}{1+\dfrac{s}{50}}$$





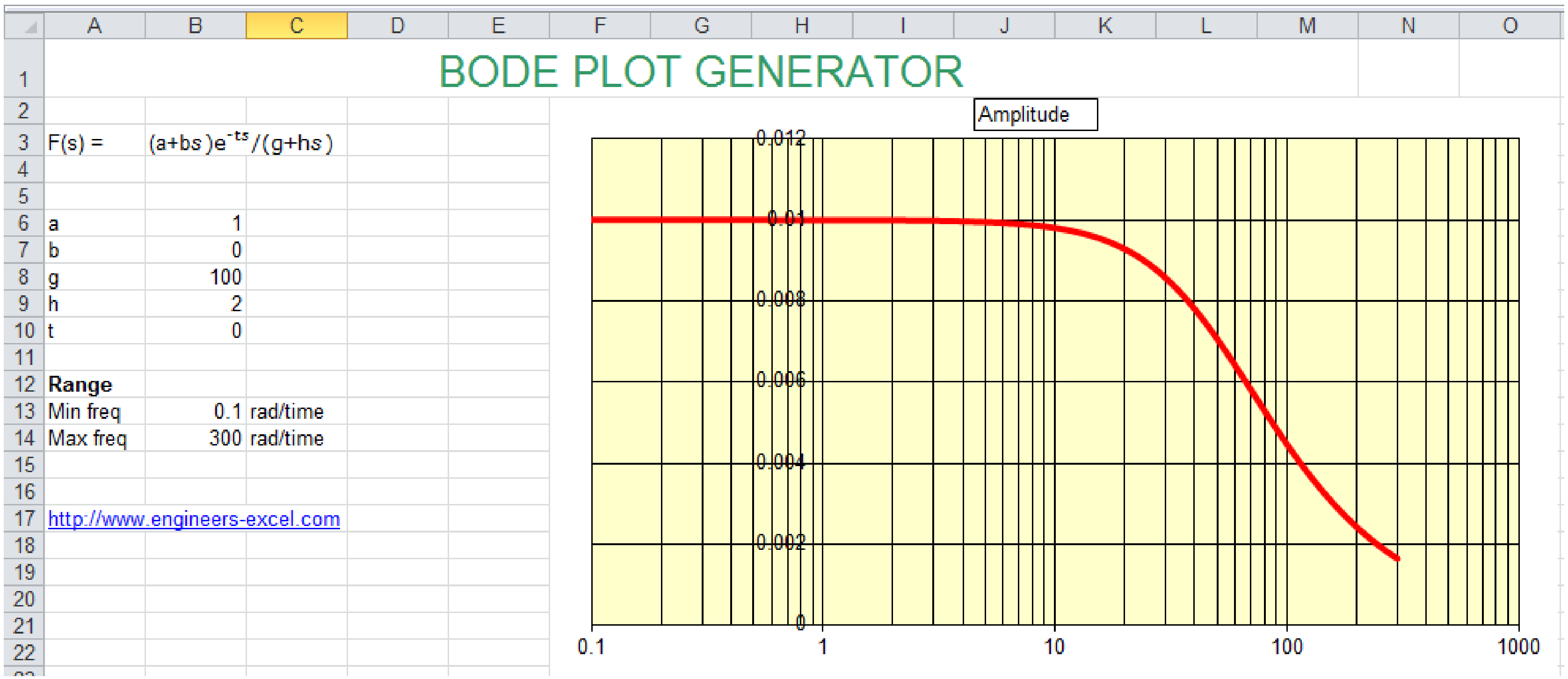
FILTERS – Transfer Functions/Bode Plots

BODE Plot of a transfer function

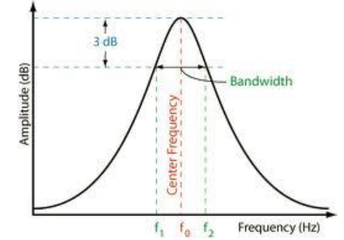
$$|H(s)| = \frac{1}{2s+100}$$

$$|H(s)| = \frac{a+bs}{g+hs}$$

$$a = 1, b = 0, g = 100, h = 2$$



FILTERS – Transfer Functions/Bode Plots



Let: L = 100 mH, C = 10 mF, R = 11 Ω

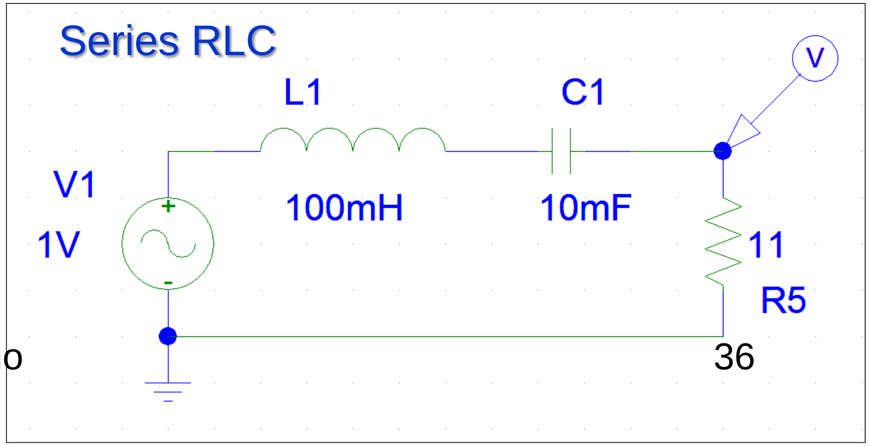
$$|H(s)| = \frac{s\dfrac{R}{L}}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} = \frac{110s}{s^2 + 110s + 1000}$$

$$|H(s)| = \frac{s\dfrac{\omega_o}{Q_o}}{s^2 + \dfrac{\omega_o}{Q_o}s + \omega_o^2}$$

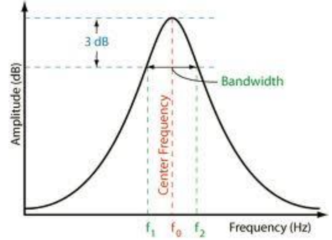
BPF

$$|H(s)| = \frac{110s}{(s+10)(s+100)} = \frac{110s}{10\left(1+\dfrac{s}{10}\right)100\left(1+\dfrac{s}{100}\right)}$$

$$|H(s)| = \frac{0.11j\omega}{\left(1+j\dfrac{\omega}{10}\right)\left(1+j\dfrac{\omega}{100}\right)}$$



FILTERS – Transfer Functions/Bode Plots

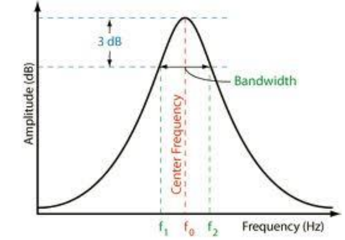


$$\left|H(j\omega)\right| = \frac{0.11 j\omega}{\left(1 + j\dfrac{\omega}{10}\right)\left(1 + j\dfrac{\omega}{100}\right)}$$

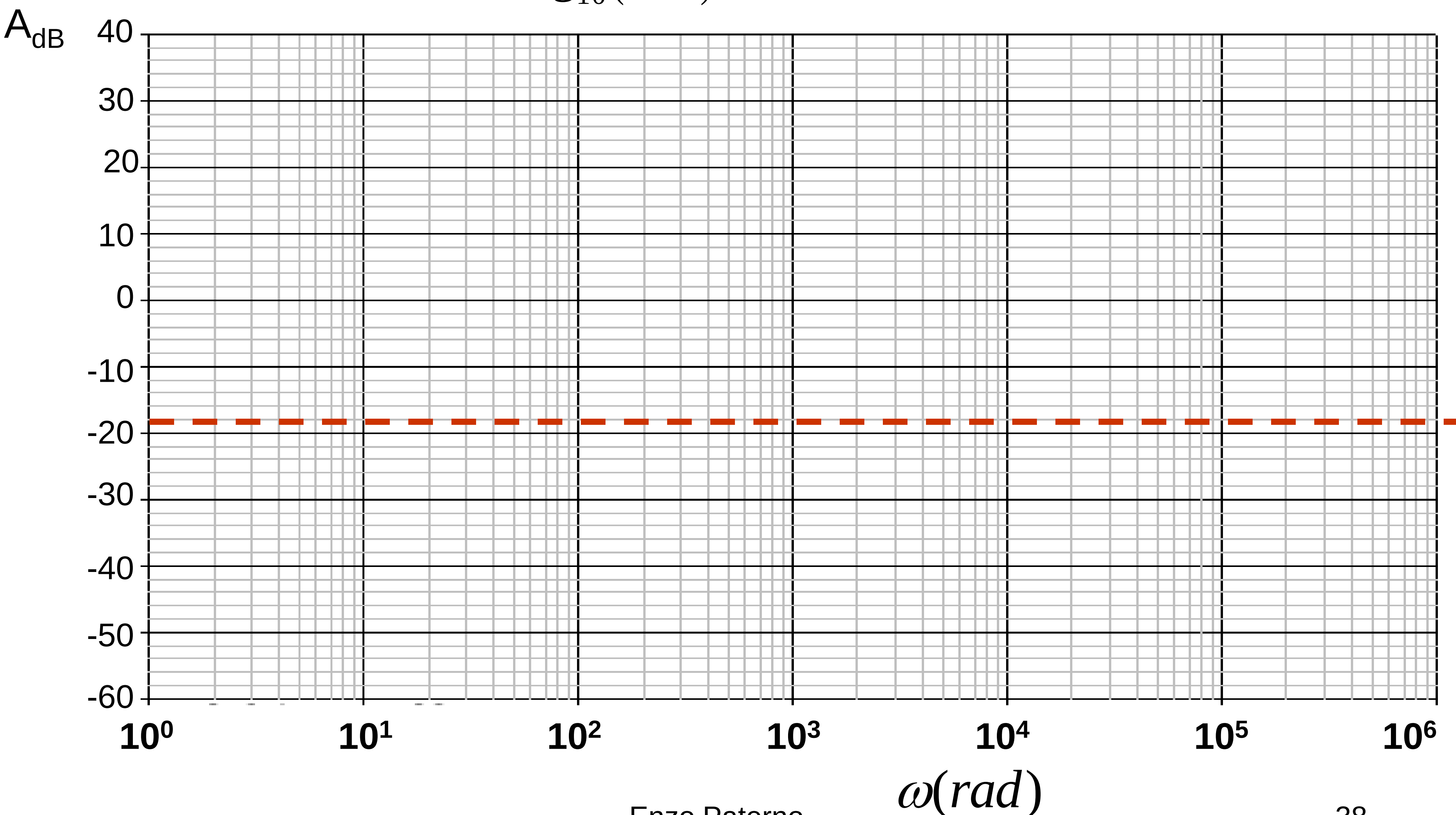
$$A_{dB} = 20\log_{10}\left|H(j\omega)\right|$$

$$A_{dB} = 20\log_{10}(0.11) + 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1 + j\frac{\omega}{10}\right| - 20\log_{10}\left|1 + j\frac{\omega}{100}\right|$$

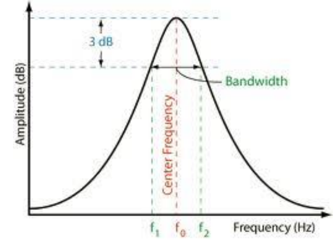
FILTERS – Transfer Functions/Bode Plots



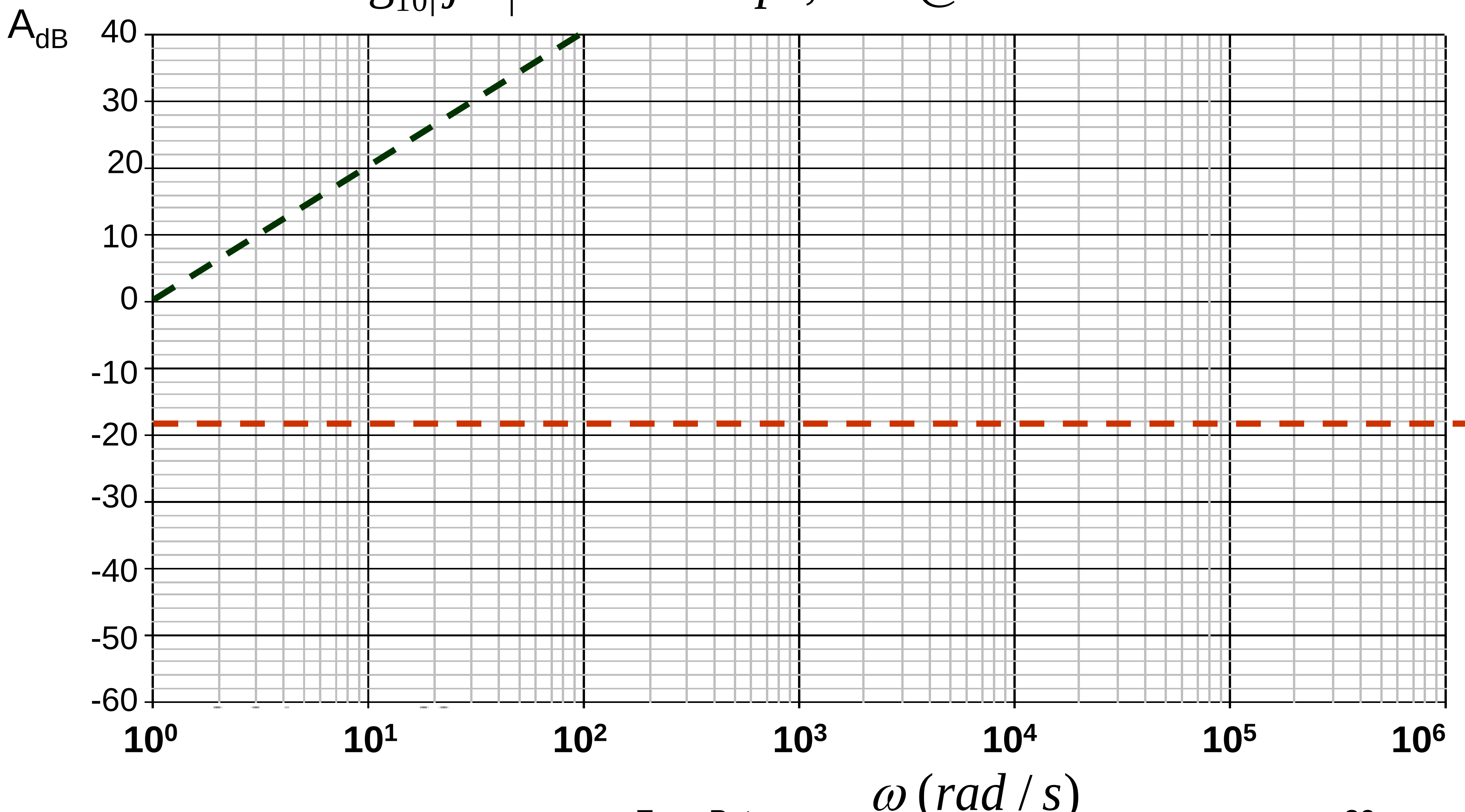
$$20\log_{10}(0.11) = -19dB$$

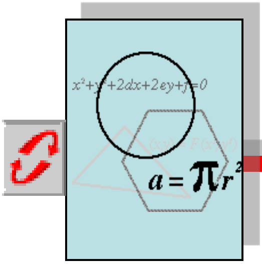


FILTERS – Transfer Functions/Bode Plots

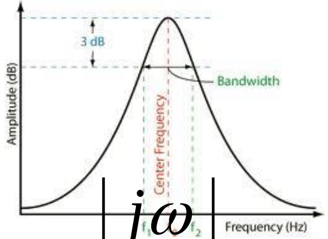


$$20\log_{10}\left|j\omega\right| \rightarrow 20db\,slope,\ 0dB@\omega = 1$$

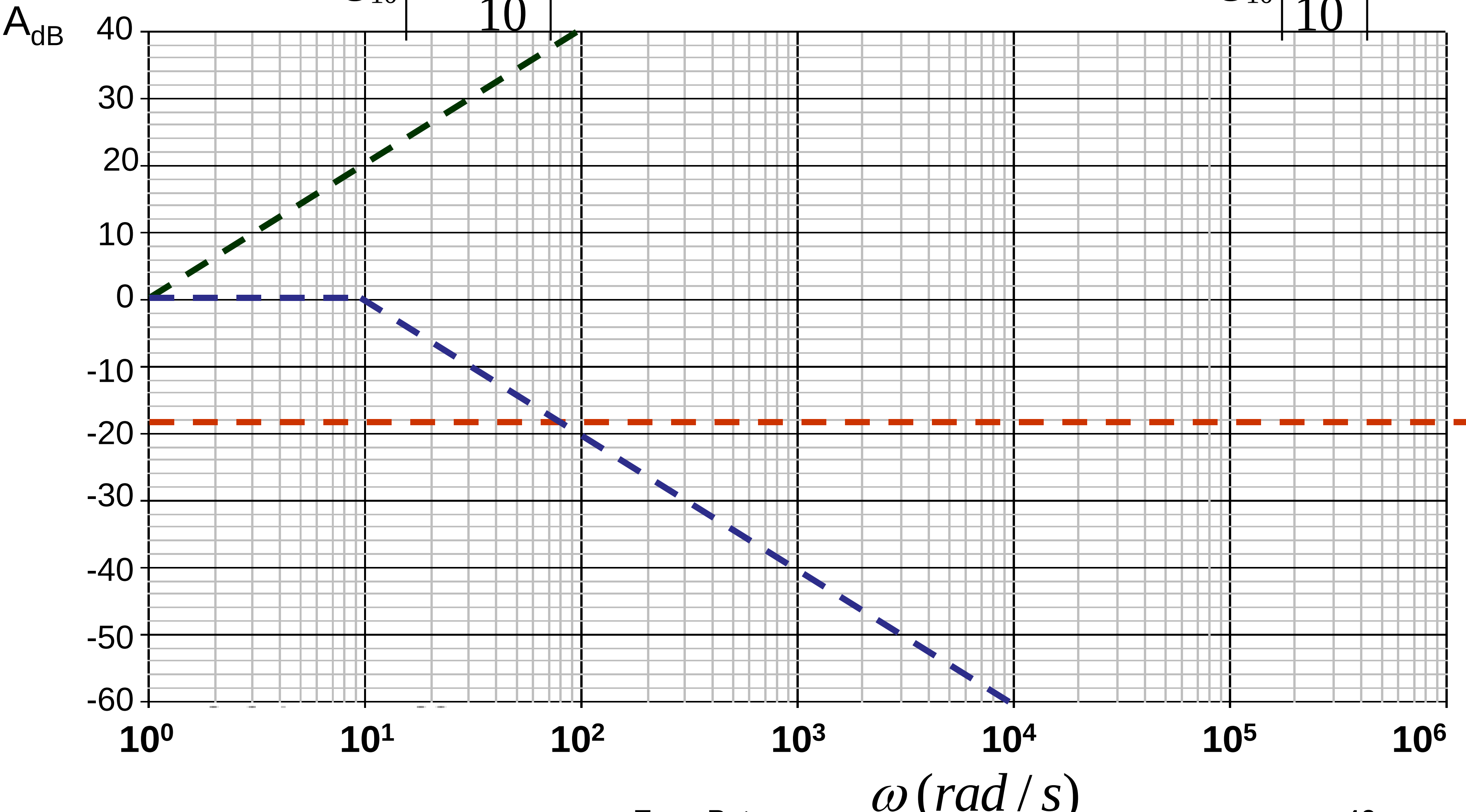




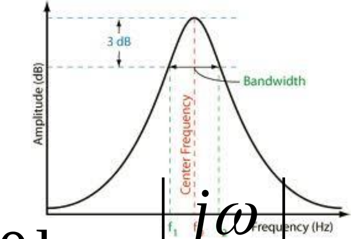
FILTERS – Transfer Functions/Bode Plots



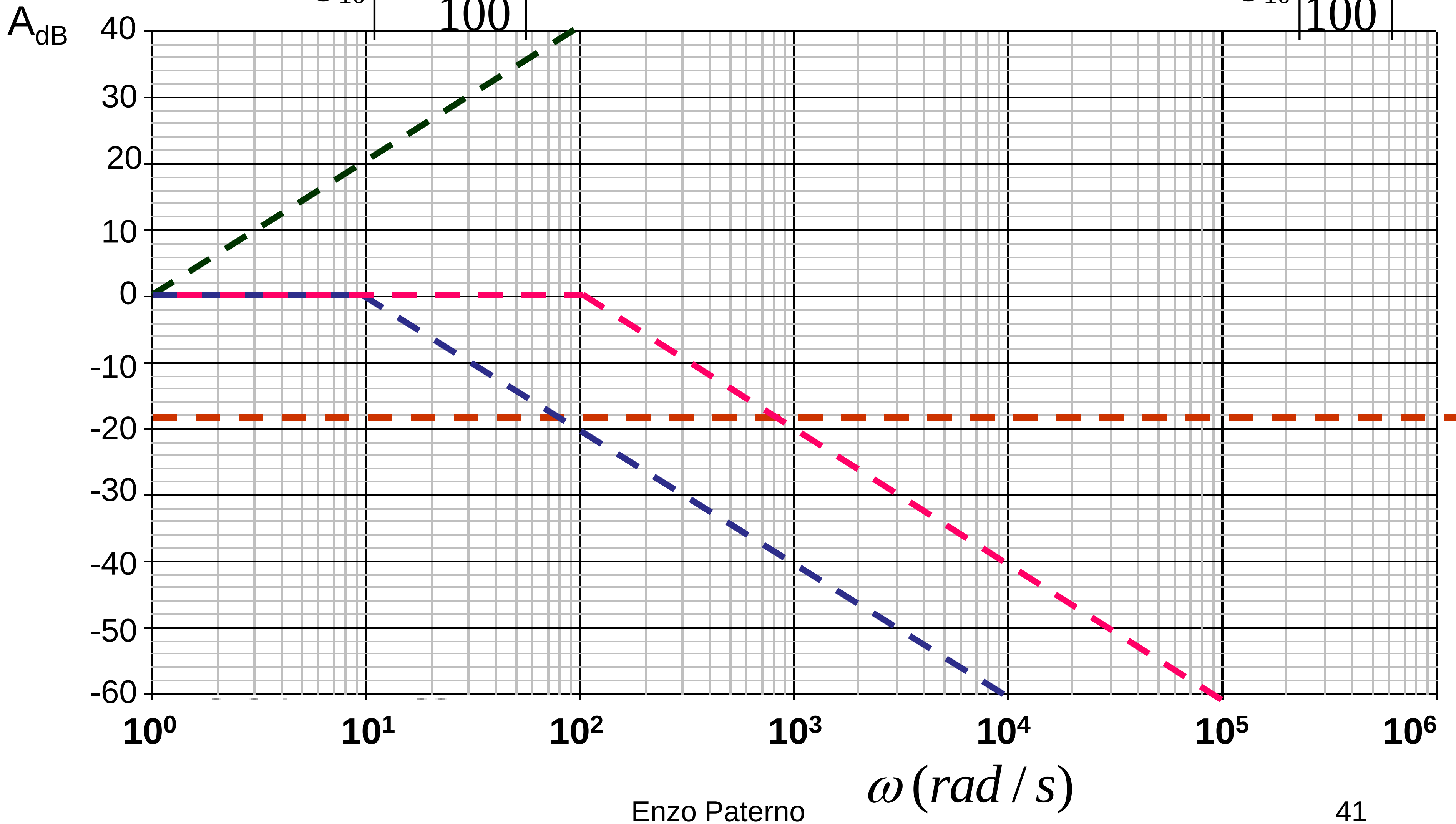
$$-20\log_{10}\left|1+\frac{j\omega}{10}\right|; \quad \omega<10 \rightarrow 0db,\ \omega>10 \rightarrow -20\log_{10}\left|\frac{j\omega}{10}\right|$$



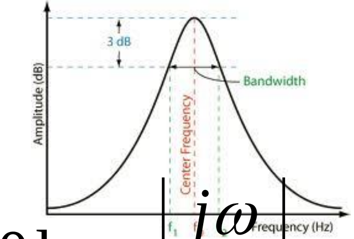
FILTERS – Transfer Functions/Bode Plots



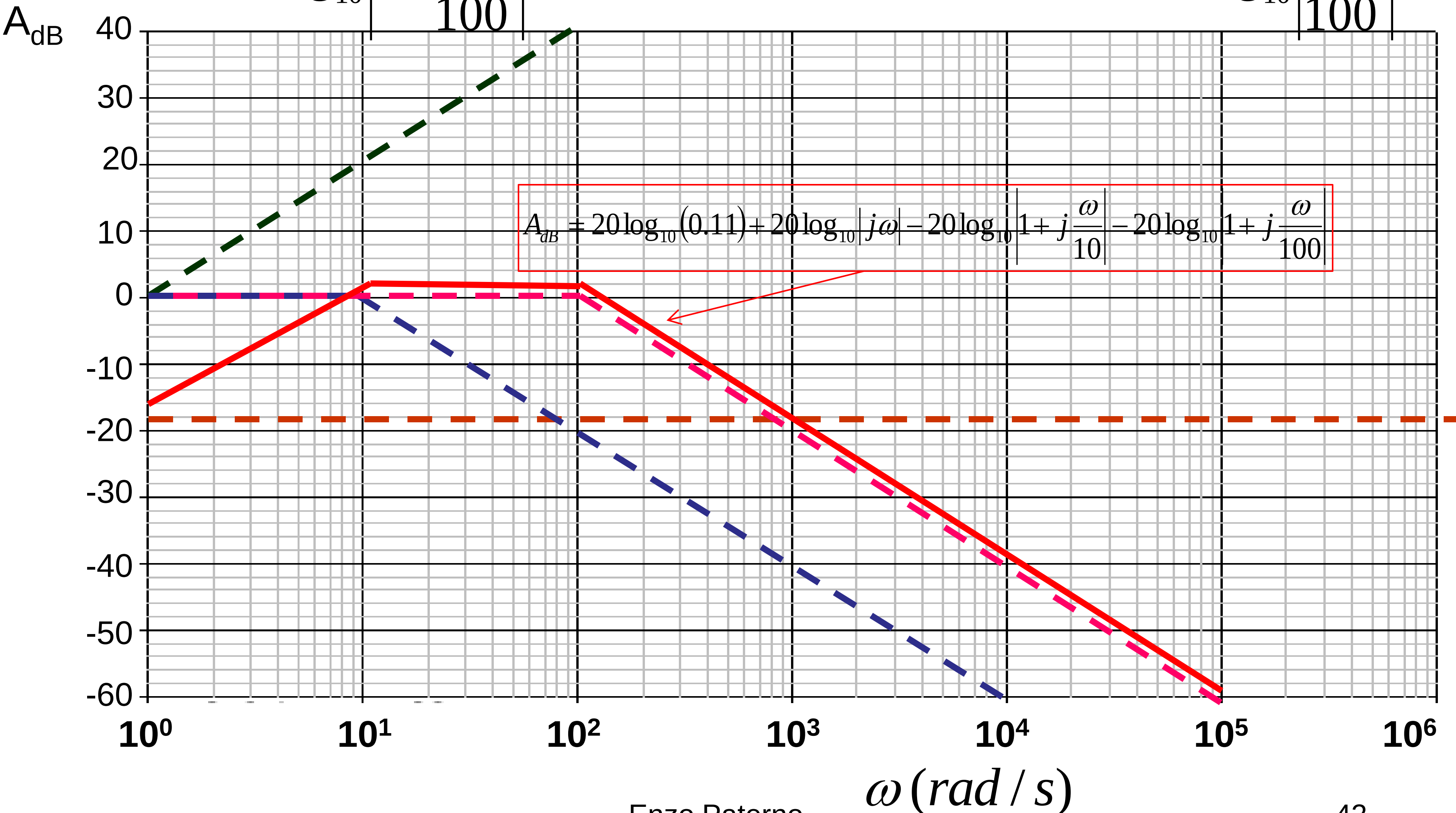
$$-20\log_{10}\left|1+\frac{j\omega}{100}\right|; \quad \omega<100 \to 0db,\ \omega>100 \to -20\log_{10}\left|\frac{j\omega}{100}\right|$$

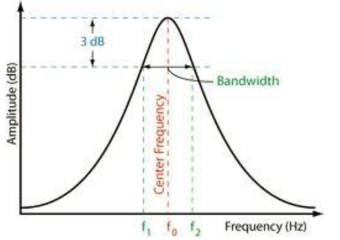


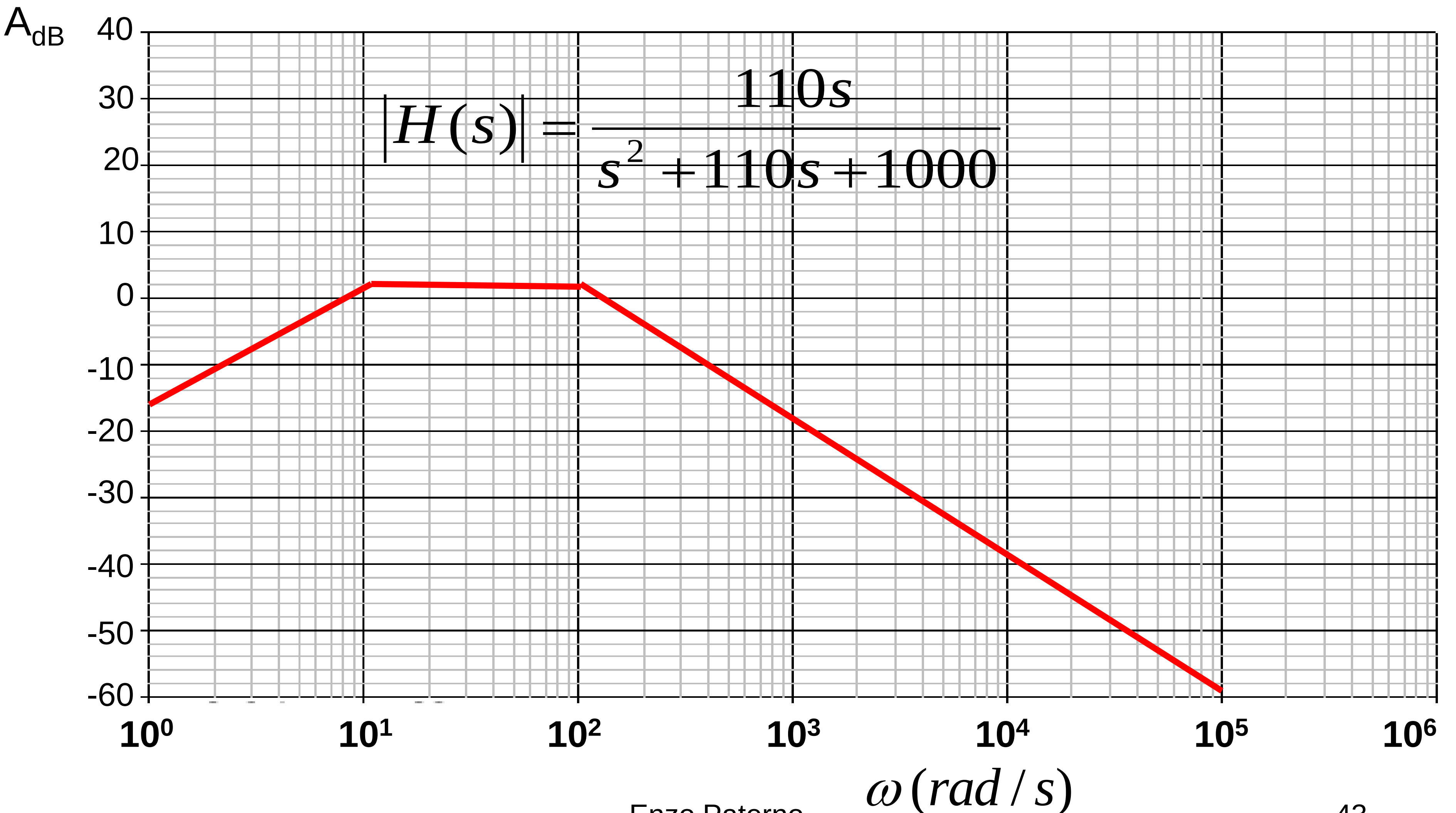
FILTERS – Transfer Functions/Bode Plots



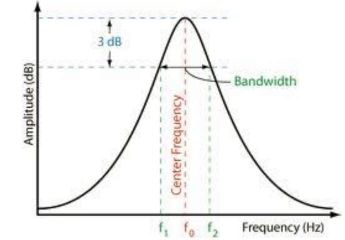
$$-20\log_{10}\left|1+\frac{j\omega}{100}\right|;\quad \omega<100\rightarrow 0db,\ \omega>100\rightarrow -20\log_{10}\left|\frac{j\omega}{100}\right|$$




3 dB
Bandwidth
Amplitude (dB)
Center Frequency
Frequency (Hz)


A_{dB}
40
30
20
10
0
-10
-20
-30
-40
-50
-60
$|H(s)| = \frac{110s}{s^2 + 110s + 1000}$
10^0
10^1
10^2
10^3
10^4
10^5
10^6
$\omega\,(rad/s)$

FILTERS – Transfer Functions/Bode Plots



$$|H(s)| = \frac{110s}{s^2 + 110s + 1000}$$

H(s) = (a0+a1s+a2s2+a3s3+a4s4+a5s5) / (b0 + b1s+b2s2+b3s3+b4s4+b5s5)

0 110 0 0 0 0 1000 110 1 0 0 0

BODE PLOT GENERATOR

F(s) = $(a_0+a_1s+a_2s^2+a_3s^3+a_4s^4+a_5s^5)/(b_0+b_1s+b_2s^2+b_3s^3+b_4s^4+b_5s^5)$

a_0	0	b_0	1000
a_1	110	b_1	110
a_2	0	b_2	1
a_3	0	b_3	0
a_4	0	b_4	0
a_5	0	b_5	0

Range:

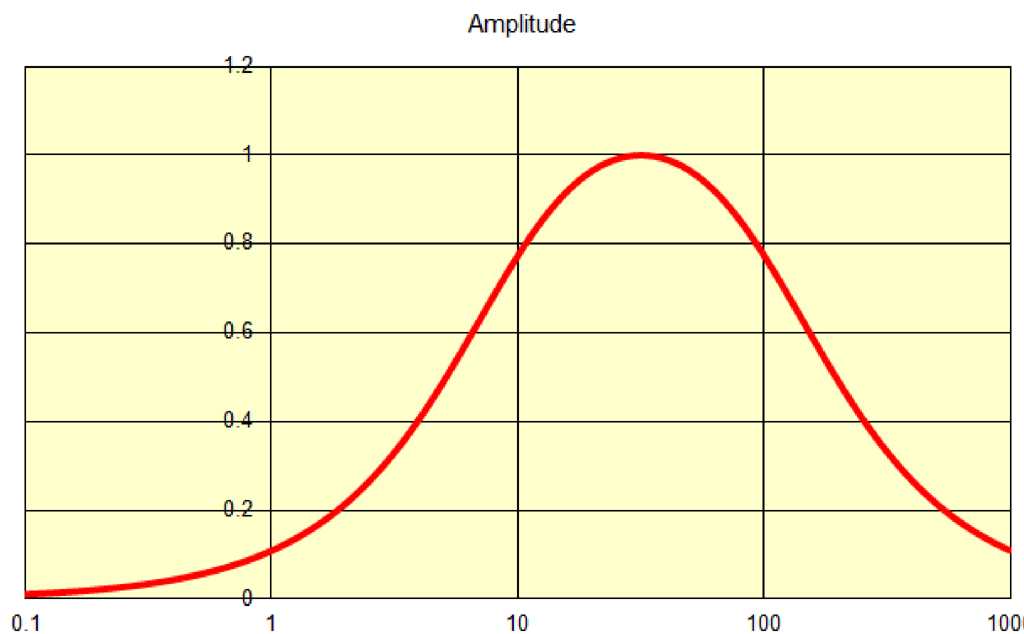
Min freq 0.1 rad/time

Max freq 1000 rad/time

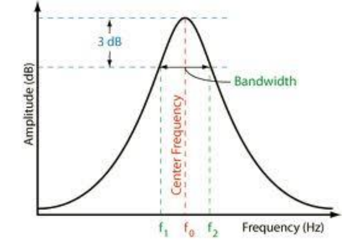
Note:

This tool requires the Analysis ToolPack add-in.

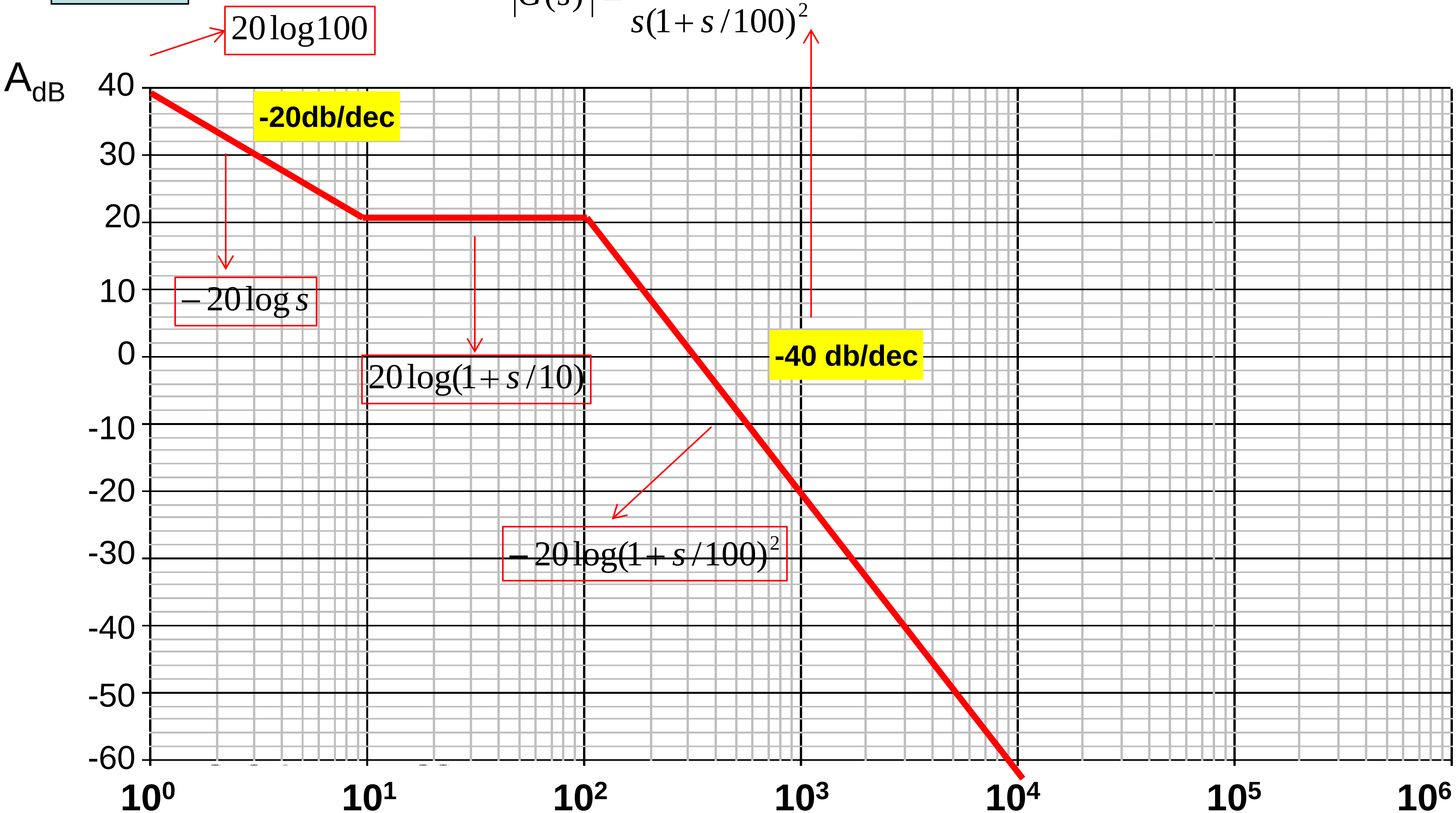
http://www.engineers-excel.com



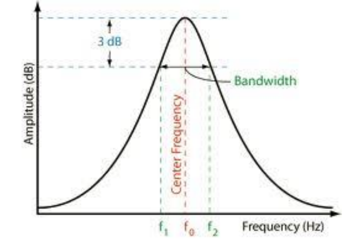
BODE PLOT - Example

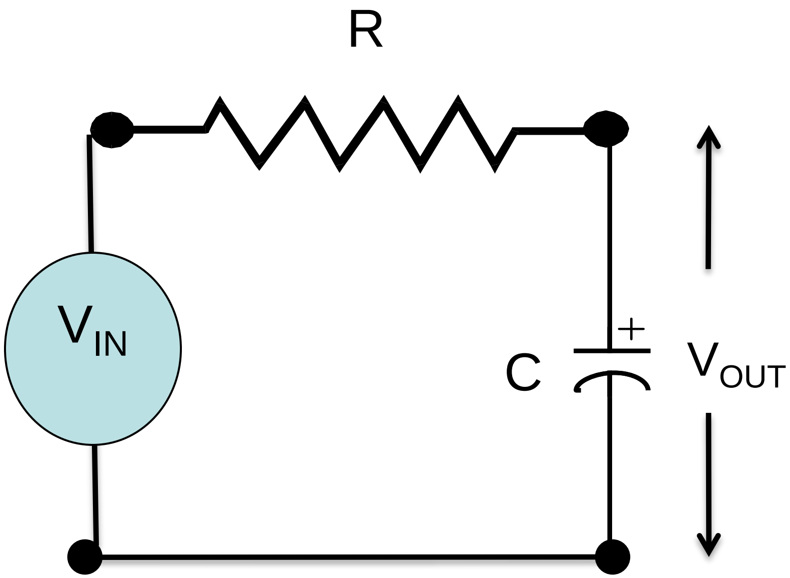


$$|G(s)| = \frac{100(1+s/10)}{s(1+s/100)^2}$$



LPF FREQUENCY RESPONSE

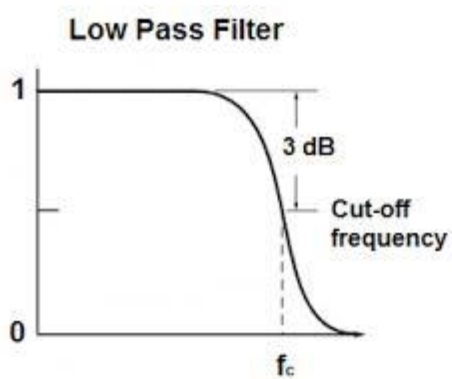




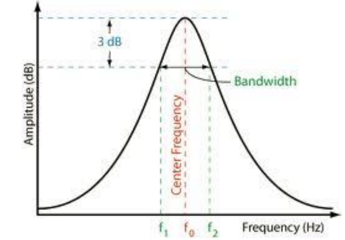
$$V_{OUT} = \frac{V_{IN}(-jXc)}{R - JXc}$$

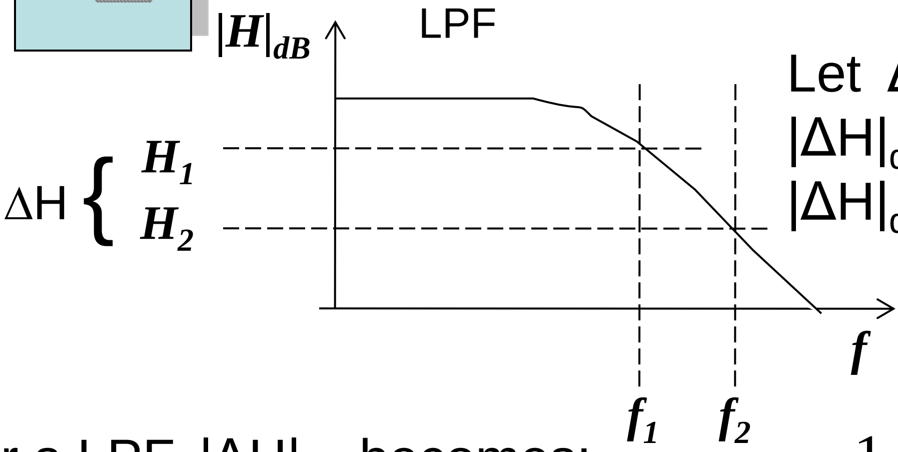
$$|H(j\omega)| = \frac{V_{OUT}}{V_{IN}} = \frac{Xc}{\sqrt{R^2 + X^2c}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \frac{R^2}{X^2c}}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$



FILTERS – ROLL OFF





Let $\Delta H = H_2 - H_1$

$|\Delta H|_{dB} = 20 \log_{10} |\Delta H|$

$|\Delta H|_{dB} = 20 \log_{10} |H_2| - 20 \log_{10} |H_1|$

$$\therefore \left|\Delta H\right|_{dB} = 20\log_{10}\frac{|H2|}{|H1|}$$

For a LPF, $|\Delta H|_{dB}$ becomes:

$$\left|\Delta H\right|_{dB} = 20\log_{10}\beta \quad \text{with } \beta = \frac{\dfrac{1}{\sqrt{1+(\omega_2 RC)^2}}}{\dfrac{1}{\sqrt{1+(\omega_1 RC)^2}}}$$

We simplify β:

$$\beta = \frac{\sqrt{1+(\omega_1 RC)^2}}{\sqrt{1+(\omega_2 RC)^2}} \quad \text{assume } \omega RC >>>> 1$$

$$\therefore \omega >>>> \frac{1}{RC}$$

$$\left|\Delta H\right|_{dB} = 20\log_{10}\frac{\omega_1}{\omega_2}$$

$$\boxed{@\,\omega_2 = 2\omega_1 \mapsto \left|\Delta H\right|_{dB} = -6dB,\ @\,\omega_2 = 10\omega_1 \mapsto \left|\Delta H\right|_{dB} = -20dB}$$