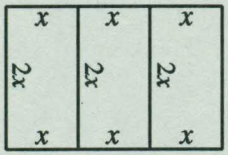
Contest # 1 — *Answers & Solutions* — 10/15/13

Problem 1-1

The perimeter of each rectangle is 18, so $6x = 18$, and $x = 3$. The dimensions of each rectangle are 3 and 6, so the area of each is $3 \times 6 = \boxed{18}$.



Problem 1-2

The greatest common factor of the two chosen numbers is 24, so each must be a multiple of 24. The largest multiples of 24 that are less than 100 are $3 \times 24 = 72$ and $4 \times 24 = 96$. Their greatest common divisor is 24, and their sum is $3 \times 24 + 4 \times 24 = 7 \times 24 = \boxed{168}$.

Problem 1-3

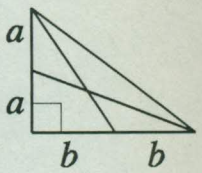
Since $BBB/3 = ABC$ is a 3-digit number that retains B as its middle digit and changes the other 2 digits, we can rule out $BBB = 333$, 666, or 999. Now try each of $BBB = 111$, 222, 444, 555, 777, and 888. The only solution of $ABC = BBB/3$ comes from $ABC = 444/3 = 148$, so $(A,B,C) = \boxed{(1,4,8)}$.

Problem 1-4

The number 10^{2013} has 2014 digits, 2013 of which are 0s. The result of the subtraction $10^{2013} - 2013$ is $1000\ldots00 - 2013 = 999\ldots997987$, a number with 2013 total digits. The sum of the last 4 digits is $7+9+8+7 = 31$. The first $2013 - 4 = 2009$ digits have a sum of $2009 \times 9 = 18\,081$. The sum of all 2013 digits is $31 + 18\,081 = \boxed{18\,112}$.

Problem 1-5

In the diagram, $a^2+4b^2 = 4^2$ and $4a^2+b^2 = 3^2$. Adding, we get $5a^2+5b^2 = 16+9 = 25$. Dividing both sides by 5 and then multiplying both sides by 4, we get $4a^2+4b^2 = 20 =$ hypotenuse^2, so hypotenuse's length is $\boxed{\sqrt{20}}$.



Problem 1-6

The 1 bad surfboard in 1000 will almost always test bad. Of the 999 good surfboards in 1000, 1% (about 10) typically test bad, even though they are good surfboards. Of the 11 surfboards that test bad, the expected number of bad surfboards is 1, and 1/11 is about $\boxed{9\%}$.